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New chaotic patterns in pulsed discharges

Min Sup Hur, Jae Koo Lee *, Bong Koo Kang

Department of Electrical Engineering, Pohang University of Science and Technology, Pohang 790-784, South Korea

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Abstract

We observed new chaotic patterns in discharge spikes of a pulsed plasma system having parallel electrodes and dielectric barriers. The chaotic behavior was modeled by computer-intensive simulations and a separate semi-analytic map, showing that the periodic patterns transit to chaos without standard cascades such as period doubling, quasiperiodicity, or intermittence.
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Nonlinear dynamics of plasma have been the subjects of many researches. Temporal or spatio-temporal chaos have been observed and intensive studies in many systems such as beam plasmas [1–5], double plasmas [6], coaxial discharge systems [7], and dielectric barrier discharges [8] have been carried out. Most of these plasmas are energy-dissipative systems and show universal characteristics; period doubling, quasiperiodicity, intermittence, spatio-temporal chaos, and pattern formation. And a common parameter was found to describe the chaos in beam-plasma systems [3–5].

In this letter, we report a different kind of nonlinear dynamics in a dielectric barrier discharge (DBD) driven by voltage pulses. Since the electrical breakdown occurs only in the pulse-on duration, spikes of discharge currents appear periodically with the same frequency as the applied pulses. We observed that the height of the spikes becomes chaotic when a special kind of pulse shape is used. The cascades to chaos are different from the standard routes such as period doubling, quasiperiodicity, or intermittence. The spike pattern is NP when the same pattern is repeated every N pulses. The usual period doubling cascade shows sequence of $1P \to 2P \to 4P \to \cdots \to$ Chaos. However, from the fluid simulation of DBD, we observed the sequence of $1P \to$ Chaos $\to 2P \to$ Chaos $\to 3P \to$ Chaos ... as a control parameter was changed. From detailed analysis of these phenomena, we obtained a semi-analytic map which relates two successive discharge spikes. The analysis of the map showed that the periodic patterns with very high N are densely mixed in the parameter regimes of chaos.

The simulations were performed for a one-dimensional system with two planar electrodes and dielectric barriers in front of the electrodes. We employed continuity and drift-diffusion equations [9] of electrons and Ne ions with the Poisson equation. The parameters used in the simulations are for a one-dimensional two-electrode system of gap size 0.02 cm covered with

* Corresponding author.
E-mail address: jkd@postech.ac.kr (J.K. Lee).

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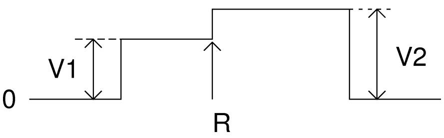

Fig. 1. Two-tiered pulse used in the simulation. The total pulse width is 5 μs. The repetition frequency of the pulse is 0.125 MHz. V_1 and V_2 are the voltages of two tiers, and R is the beginning of the second tier.

dielectric materials of secondary electron emission coefficient 0.2 with pure Ne gas at 300 Torr pressure. The pulse with two tiers (Fig. 1) is applied to one of the electrodes with alternating signs. The pulse duration is 5 μs and it is repeated with the frequency of 0.125 MHz.

The physical variables to characterize the discharge at each pulse are height of the spike, discharge ignition time from the beginning of the pulse (s), and wall voltage due to the charges accumulated on the dielectric surfaces (V_w). All these characteristic values are determined by the voltage difference between the gap of two dielectric layers (V_g). If V_g is determined only by the applied voltage, the discharge spikes will be constant giving no more interesting dynamics. Because the wall voltage accumulated during a pulse influences the next V_g, the discharge spike is dependent not only on the applied voltage but also on the previous discharge. Therefore there exists a map which connects the physical variables characterizing a discharge in a pulse to those in the next pulse. The behaviors of this map can be studied conveniently by constructing a return map. Our simulations show little difference in the discharge peaking time and the discharge ignition time in most cases, so that the discharge peaking times are taken as the approximations to the discharge ignition times when constructing the return map. The error is less than 10% in most cases. Fig. 2 shows the return maps of (s^n, s^{n+1}) and the corresponding power spectra obtained from the simulation. s^n is the discharge ignition time from the beginning of n'th pulse. The periodic and stochastic patterns appear alternatively as V_1 (Fig. 1) is decreased. The noise levels in the power spectra of stochastic patterns are relatively high. This implies that they are chaotic.

For more detailed analysis, we constructed a semi-analytic model for the system. The gap voltage at the beginning of $(n+1)$th pulse is calculated as follows:

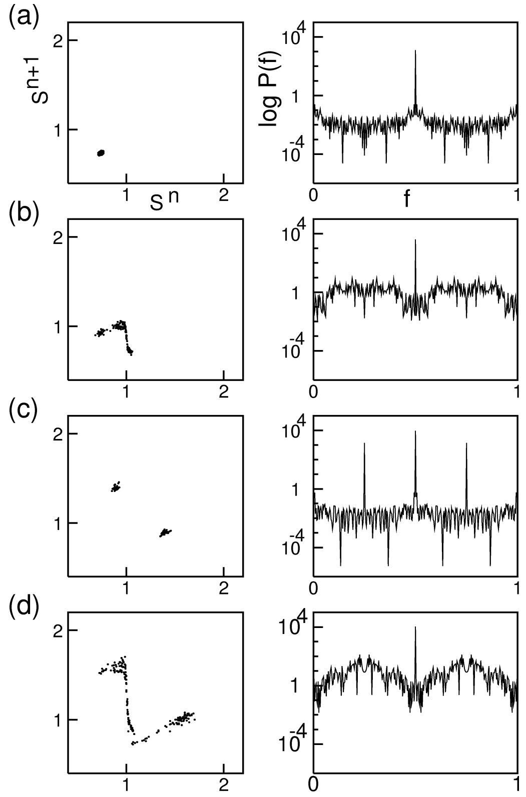

Fig. 2. Return maps of (s^n, s^{n+1}) and the corresponding power spectra obtained from the simulation, where s^n is the discharge ignition time (μs). The values of V_1 are (a) 140.0 V, (b) 138.7 V, (c) 136.0 V, and (d) 134.0 V. V_2 and R are fixed as 142 V and 1 μs, respectively. The main frequency in the power spectra is 0.125 MHz which is the repetition rate of the pulses.

$$V_g^{n+1} = f V_a + V_w^n, \tag{1}$$

where V_a is the applied voltage, V_w^n is the wall voltage at the end of the nth pulse, and f is a reduction factor of the applied voltage due to dielectric layers. Because the $(n+1)$th discharge is uniquely determined by V_g^{n+1}, there exists a one-dimensional map connecting V_w^n to V_w^{n+1}. The analytic form of this map can be obtained by following procedures. The functions S and T are defined as

$$s = S(V_g), \tag{2}$$

$$w = T(V_g), \tag{3}$$

where s is the discharge ignition time and w is the wall voltage accumulation due to V_g. The function $T(V_g)$ is called a voltage transfer function, which represents how much wall voltage is accumulated on the dielectric layers after a discharge by V_g is terminated. w is called transferred voltage. Then V_w^{n+1} is a difference between the transferred voltage by $(n+1)$th discharge and the previous wall voltage V_w^n, since the wall charges are accumulated over the previous ones with an opposite sign. Therefore, using Eqs. (1) and (3), V_w^{n+1} is calculated as

$$V_w^{n+1} = T\left(V_w^n + fV_a\right) - V_w^n. \tag{4}$$

As in Fig. 1, V_a is dependent on the discharge ignition time. Using Eq. (2) and from Fig. 1, the following equation is obtained:

$$V_a = \begin{cases} V_1 & \text{when } S(V_w^n + V_1) < R, \\ V_2 & \text{when } S(V_w^n + V_1) > R, \end{cases} \tag{5}$$

where R is the beginning of the second tier as in Fig. 1. The analytic forms of S and T were found from fitting simulation data. The fitting formulae were obtained from simple analysis of discharges. The results are

$$s = S(V_g) = \frac{309}{V_g^2} - \frac{218.4}{V_g} + 38.82 \tag{6}$$

and

$$w = T(V_g) = -0.81V_g^2 + 5.81V_g - 7.12, \tag{7}$$

where the units of s and V_g are μs and $\times 10^2$ V, respectively. The unit of w is the same as that of V_g.

Eqs. (4)–(7) completely describe map M which is defined as $V_w^{n+1} = M(V_w^n)$. Fig. 3 is a plot of Eq. (4) for $fV_a = fV_1 = 120$ V (curve 1) and $fV_a = fV_2 = 135.24$ V (curve 2). The point Q represents wall voltage where the discharge ignition time $S(Q + fV_1)$ equals to R $(= 2$ μs). While V_w is smaller than Q, M is described by curve 2 and V_w approaches to P (Fig. 3). This is a stable equilibrium, as slope of the curve is less than one. When V_w becomes larger than Q, M is described by curve 1 which implies that V_w is reduced abruptly from curve 2 to curve 1. Then V_w returns to low value region and starts to increase again to the equilibrium (P). The analytic form of M agrees qualitatively with the simulation results. Fig. 4 is the return map of (V_w^n, V_w^{n+1}) and the corresponding time trace of V_w obtained from the simulation with same parameters as in Fig. 3. The upper and lower bunches of the points correspond to curve 2 and curve 1 in Fig. 3, respectively. Fig. 4(b) clearly shows slow increasing and abrupt decreasing of V_w as expected from the analysis of Fig. 3.

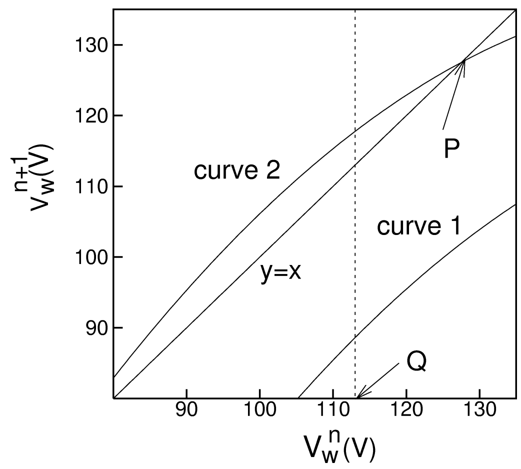

Fig. 3. The plot of Eq. (4) for $fV_a = fV_1 = 120$ V (curve 1) and $fV_a = fV_1 = 135.24$ V (curve 2).

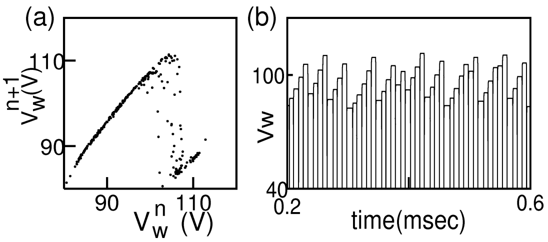

Fig. 4. (a) Return map obtained from simulation. (b) Corresponding time trace of V_w. Same values of V_1, V_2, and R $(= 2$ μs) were used as those in Fig. 3.

The slow increasing and abrupt decreasing process, represented in Figs. 3 and 4, is similar to the stretching and folding of the horseshoe map [10], showing interesting dynamics. Fig. 5 is a mode transition diagram obtained from varying V_1 with fixed values of V_2 $(= 136.64$ V) and R $(= 2.12$ μs). The pattern of $1P$ in the regime of high V_1 transits to stochastic pattern. Period doubling or quasiperiodicity could not be found between $1P$ and stochastic pattern up to the V_1 resolution of 2×10^{-5}. Periodic windows of $2P$, $3P$, $4P$, and periodic pattern with higher N appear in the middle of the stochastic regimes. Fig. 6 represents magnification of Fig. 5 near the $1P$ and $2P$ periodic

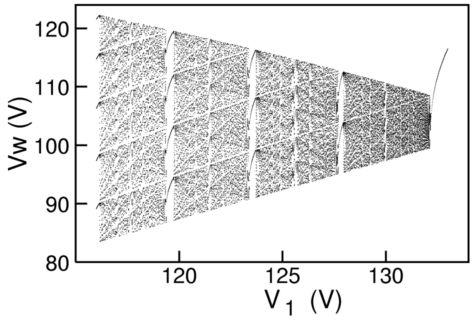

Fig. 5. Mode transition diagram for the variation of V_1 with fixed V_2 ($= 136.64$ V) and R ($= 2.12$ μs).

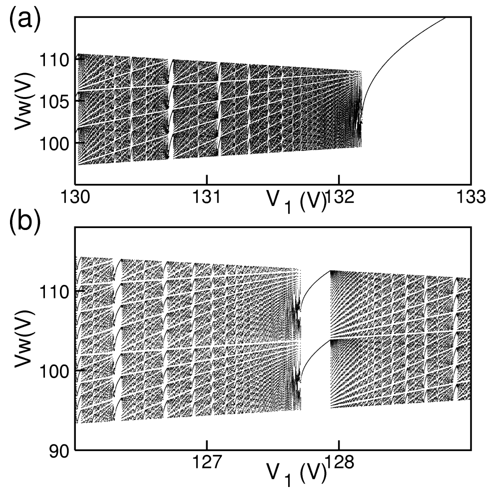

Fig. 6. Mode transition diagrams near $1P$ and $2P$ periodic windows.

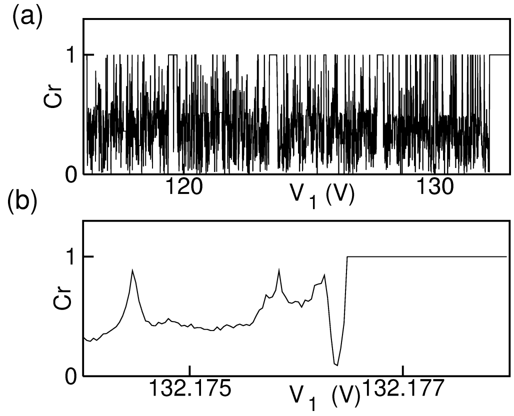

Fig. 7. (a) Correlation values for the mode transition diagram of Fig. 5. (b) Magnification of (a) near the transition from $1P$ to Chaos.

windows. It is seen that a lot of periodic states are intervened in the middle of the stochastic patterns. Fig. 6 also shows self similarity that is the typical property of nonlinear systems. Most discharge peaks occur during only one phase with V_1 or V_2 and rarely near R. These peaks are observed between two main point (upper and lower) bunches in the return map.

To measure the stochasticity of the system, correlation values were calculated. Because of the discontinuity near the point Q in Fig. 3, it is difficult to calculate the Lyapunov exponent [11]. Instead the correlation between two sequences of V_w^n with slightly different initial values provides some information about the sensitive dependence on the initial conditions. The correlation value of two signals s_1 and s_2 is

$$Cr = \frac{\langle s_1 s_2 \rangle}{\sqrt{\langle s_1^2 \rangle \langle s_2^2 \rangle}}, \tag{8}$$

where $\langle\ \rangle$ represents time-average. Fig. 7(a) represents the correlations of two V_w^n sequences with slightly different initial values. The chaos (low Cr) and the periodic (high Cr) orbits are densely mixed in the V_1-parameter space. Fig. 7(b) is the magnification of Fig. 7(a) near the transition from $1P$ to the stochastic pattern. The abrupt decrease of Cr implies that there is no multi-periodic state such as $2P$, $4P$, or quasiperiodicity between $1P$ and chaos.

In summary, nonlinear behavior different from that in other plasma systems was observed in simulation and a semi-analytic modeling of dielectric barrier discharges (DBD). Because of the wall charge accumulation, the discharges in two successive pulses are related by a one-dimensional map. As a characteristic variable to represent a discharge, the wall voltage V_w was chosen and the analytic form of the map (M) which gives the relation between V_w^n and V_w^{n+1} was found. As the control parameter V_1 is changed, the cascade of $1P \to$ Chaos $\to 2P \to$ Chaos … was observed, which is different from the standard route to chaos such as period doubling or quasiperiodicity. The mechanism of the chaos in M is summarized as ‘slow increasing and abrupt decreasing’ which is similar to the horseshoe map.

Pulse-driven DBD is widely used in many applications [12] and nonlinear behavior of the discharges is an interesting subject. In general pulse-driven discharges, two successive spikes may be related by space charges instead of the wall voltage. Cheung et al. [13] reported period doubling without the observation of clear chaos. Since the two-tiered pulse in Fig. 1 provides the stretching and folding mechanism, interesting nonlinear behaviors may be expected to occur in general pulse-driven system. The methods for finding the analytic form of the map should be applicable to other pulse-driven systems.

Acknowledgements

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