# Ion-Beam-Driven Instabilities in Bounded Dusty Plasmas

Min Sup Hur, Hae June Lee, and Jae Koo Lee

***Abstract*—The ion-beam dynamics is simulated for a bounded dusty plasma with the immobile dust particles. Since there is no dust motion, the growth of an instability is induced dominantly from the Pierce instability, but the beam motion is also greatly influenced by the Debye-shielding thermal electrons whose densities are determined from the Boltzmann relation. The analytic calculation of a dispersion relation shows the nonexistence of a uniform equilibrium which is confirmed by simulations. A period-doubling route to chaos is observed and the various nonlinear oscillations are classified in view of a diagnostic parameter which is applied to many other beam systems.**

***Index Terms*—Chaos, dusty plasma, instability, ion beam, simulation.**

## I. Introduction

THE INSTABILITY and the nonlinear oscillations in bounded beam plasmas are the concern of capacitively coupled plasmas driven by a beam or an applied voltage. These have been studied using analytic theories [1]–[6], experiments [7], [8], and simulations [9]–[11]. One possible model of bounded beam systems is the extended Pierce plasma diode which consists of ac–dc driven planar electrodes, where an electron beam goes through an immobile ion background. The analytic theories about this model were studied in [1]–[5] and the nonlinear phenomena such as the period-doubling or the quasi-periodicity were simulated in [3]–[5] and [9]–[11]. It was shown in [10] and [11] that these oscillations can be described in a unified phase diagram and classified by a diagnostic parameter.

Recently, there has been a lot of investigation on dusty plasma because of its importance in plasma processing and space plasma. The topics of these researches are mostly the ion-dust acoustic waves, dust growing and charging, dust polarization, and dust crystallization. In many theories and simulations about acoustic waves, they usually treat the homogeneous dusty plasma. In the study of dust crystallization, the boundary effects or the beam effects can be important since that occurs usually in the presence of ion flow inside the sheath region. Dust motions in the magnetic field are also investigated [12], [13]. If any dusty plasma system is observed in a long time scale, the main aspect of study is the dynamics of heavy dusts surrounded by ions and electrons which are nearly in thermal equilibrium state. In a relatively short time scale, the ions and electrons are not in the equilibrium state. So it is interesting to study their dynamics considering the heavy particle effects.

In this paper, we studied the dynamics of an ion beam going through dusts and electrons without collision. The system is bounded by two electrodes. The ion beam is injected uniformly in time from one electrode with some constant initial velocity. The other electrode is grounded or driven by a small dc voltage. The dusts and electrons are between these two electrodes through which the ion-beam travels. A linear dispersion relation is calculated analytically and the nonlinear oscillations are simulated using the particle-in-cell (PIC) code XPDP1 [14]. The paper is organized as follows. In Section II, the model equations are described and a dispersion relation is calculated. In Section III, we simulate the nonlinear oscillations and make a phase diagram for various dynamical states. In Section IV, it is shown that these states can be classified in view of a single diagnostic parameter. In Section V, a summary and conclusions are given.

## II. Dispersion

The equations to describe the ion-beam dynamics are the continuity equation and the equation of motion, where the field is calculated from Poisson's equation. The electrons are assumed to be in thermal equilibrium, so their density follows the Boltzmann relation (we use the linearized Boltzmann relation). The dust particles are so heavy that they are assumed to be immobile. The above four equations are properly rescaled as follows,

$$\frac{\partial n}{\partial \tau} + \frac{\partial}{\partial X}(nu) = 0 \tag{1}$$

$$\frac{\partial u}{\partial \tau} + u\frac{\partial u}{\partial X} = -\frac{\partial \phi}{\partial X} \tag{2}$$

$$\frac{\partial^2 \phi}{\partial X^2} = -\alpha^2 \tilde{n} + k_{De}^2 \phi \tag{3}$$

where $\tilde{n}$ is the perturbation of rescaled ion density. The rescaling convention is

$$n = \frac{n_i}{n_{i0}}, \quad \tau = \frac{v_{i0} t}{L}, \quad X = \frac{x}{L}, \quad u = \frac{v_i}{v_{i0}}$$
$$\phi = \frac{eV}{m v_{i0}^2}, \quad \alpha = \frac{\omega_{pi} L}{v_{i0}}, \quad k_{De} = \frac{L}{\lambda_e} \tag{4}$$

where $n_{i0}$ and $v_{i0}$ are the unperturbed density and velocity of the ion-beam injected from an electrode. The unperturbed electron density $n_{e0}$ and the dust density $n_{d0}$ are rescaled

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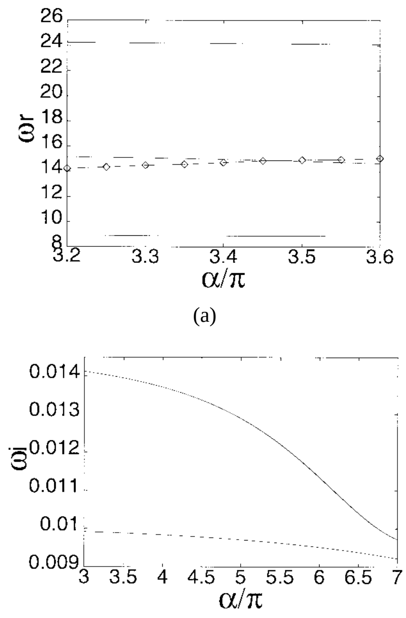



Fig. 1. The dispersion of the theory (the solid lines) and simulation results (the symbols). (a) Frequency versus $\alpha/\pi$. (b) Growth rate versus $\alpha/\pi$. $\omega_r$ and $\omega_i$ are rescaled according to (4).

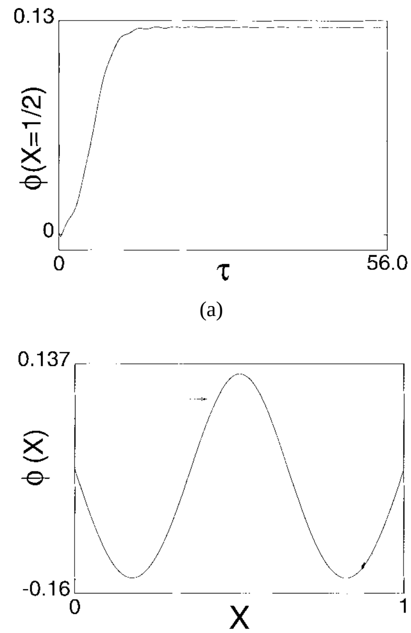



Fig. 2. The equilibrium state for $\alpha/\pi = 3.2$ and $k_{De} = 22.3$. (a) The time trace of potential at the middle point. (b) The potential profile when the system reached the equilibrium state. All the values marked in the figures are rescaled according to (4).

with respect to the $n_{i0}$. The $V$ is the potential and the $L$ is the system length. The quasi-neutrality condition is satisfied; $1 = n_{e0}/n_{i0} + Zn_{d0}/n_{i0}$. Here the $Z$ is the dust charge.

Assuming that all the perturbations have the waveform $\exp(ikX - iw\tau)$, a linear dispersion relation for the unbounded system can be derived. The result is

$$1 = \frac{\alpha^2}{(w-k)^2} - \frac{k_{De}^2}{k^2}. \tag{5}$$

When $k_{De} \gg k$, which means that the induced wavelength is much larger than the electron Debye length, (5) has two roots

$$k_{\pm} \simeq \frac{w}{1 \pm \dfrac{\alpha}{k_{De}}}. \tag{6}$$

Following the Pierce linear theory [1] with boundary conditions $n(X=0) = 1$, $u(X=0) = 1$, $\phi(X=0) = \phi(X=1) = 0$, the linear dispersion for the bounded system is derived

$$\frac{i\beta_+\beta_-(\beta_+ - \beta_-)}{k_{De}^2 - \alpha^2} w\big(k_{De}^2 - \alpha^2 - \beta_-\beta_+ w^2\big) + \beta_-^2 e^{i\beta_+ w} - \beta_+^2 e^{i\beta_- w} + \beta_+^2 - \beta_-^2 = 0 \tag{7}$$

where $\beta_{\pm} = k_{\pm}/\omega = 1/(1 \pm \alpha/k_{De})$. This system is dissipative because of the particle loss through the boundaries and is driven by the ion beam. So the ion-beam parameters such as beam density or beam velocity play an important role to make the system stable or unstable. These ion-beam parameters are reflected in $\alpha$.

Equation (7) has many solutions implying that there are as many possible modes. Fig. 1 shows some solutions of (7) for $k_{De} = 59.5$. The simulation results and the analytic solution for the linear frequency are compared in Fig. 1(a). It is not known why a special mode of perturbation is measured dominantly though the initial perturbations are arbitrary. As in Fig. 1(b), there exist growing modes for $3 \le \alpha/\pi \le 7$ which means the uniform equilibrium is always unstable in this regime. The impossibility of stable uniform equilibrium for large $k_{De}$ was verified by many simulations in the regime of $3 \le \alpha/\pi \le 4$.

Equation (7) was derived for $k_{De} > \alpha$, but Rosenberg and Shukla derived the dispersion of different form for $k_{De} < \alpha$ in [6]. In this regime, the dispersion gives a purely imaginary solution, accordingly any perturbation would grow or damp out without any oscillations.

## III. Nonlinear Oscillations

In Section II it was shown that there is no stable uniform equilibrium. Any given initial perturbations would grow and saturate to make nonlinear oscillations in some parameter regimes. Since these solutions are difficult to obtain analytically from (1)–(3), we used a PIC code [14] to simulate them. There are interesting nonlinear modes—the nonuniform equilibrium, the periodic oscillation, the chaotic oscillation, the blocking oscillation, and the dc-current mode. It depends on the input parameters $\alpha$ and $k_{De}$ to which mode the system saturates. We discuss each mode briefly and represent them in the $k_{De}$ versus $\alpha/\pi$ phase-diagram.

### A. Nonuniform Equilibrium Mode

While the uniform equilibrium is always unstable in the regime of $3 \le \alpha/\pi \le 4$, the nonuniform equilibrium is stable for the wide range of $\alpha$. Fig. 2(a) is the potential at the middle point of the system. As time advances, the initial perturbation near $\phi(X = 1/2) = 0$ grows and saturate at nonzero value. The wiggling about this nonzero value damps out quickly,

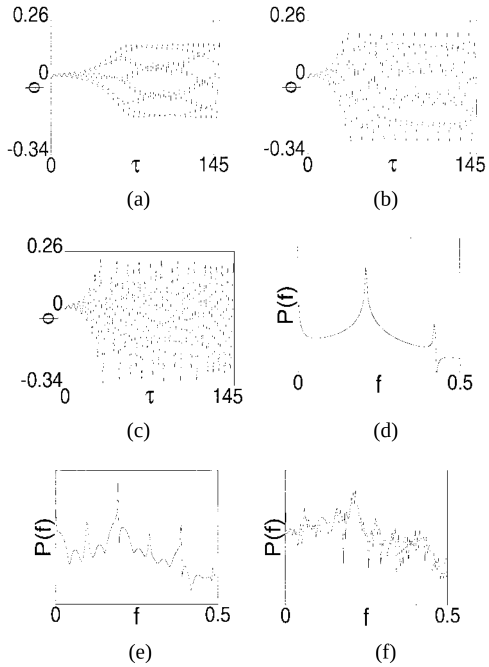



Fig. 3. Period-doubling route to chaos obtained by varying $\alpha$ for $k_{De} = 29.8$: (a) period one, $\alpha/\pi = 3.1$, (b) period two, $\alpha/\pi = 3.082$, (c) chaos, $\alpha/\pi = 3.077$, and (d)–(f) are the power spectrums of $1P$, $2P$, chaos, respectively. All the values marked in the figures are rescaled according to (4).

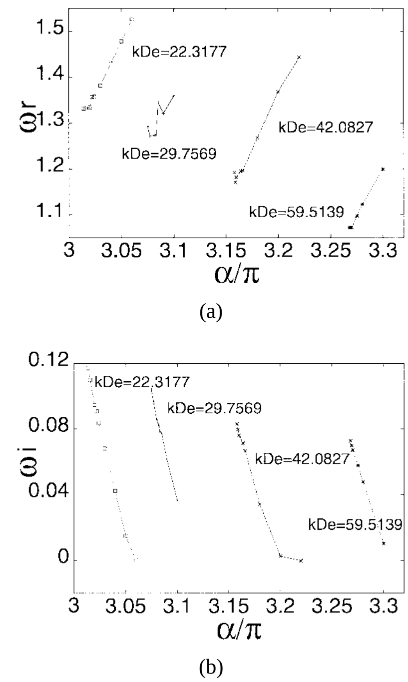



Fig. 4. (a) The frequencies of nonlinear oscillations and (b) the growth rates of perturbations about nonuniform equilibrium. All the values marked in the figures are rescaled according to (4).

which means that this nonuniform equilibrium state is stable for small perturbations. Fig. 2(b) is the potential profile for this equilibrium state.

### B. Periodic and Chaotic Oscillation Modes

In some other parameter regimes of $k_{De}$ and $\alpha$, the nonuniform equilibrium state described above becomes unstable and there appear oscillation modes. The period-one oscillation is obtained by decreasing the $\alpha$ or increasing the $k_{De}$ from the equilibrium state and the period doublings are observed as these parameters are changed furthermore (see Fig. 3). Fig. 3(d) and (e) are the power spectrums of period one and two oscillations. Clear peaks and subpeaks are seen. On the other hand, there is no such clear peaks in Fig. 3(f) and the noise level is much higher compared with Fig. 3(d) and (e) which implies that the signal in Fig. 3(c) is chaotic.

The period-doubling cascade is also observed in the Pierce plasma diode and in other beam systems [3], [9]–[11], [15]. Fig. 4 shows the frequencies of nonlinear oscillations and the growth rates obtained from simulations at various $k_{De}$ and $\alpha$. The rescaled ion-plasma-frequency is $\omega_{pi} = W_{pi}L/v_{i0} = \alpha \sim 10$. Fig. 4(a) shows that the rescaled frequencies of nonlinear oscillations are order of 1 which is one tenth of ion-plasma-frequency.

### C. DC-Current Mode

If we decrease the $\alpha$ or increase the $k_{De}$ from the chaotic oscillation state, the potential barrier [see the arrow mark in Fig. 2 (b)] becomes so high that ions in the beam start to turn backward at this potential barrier (the position of the barrier is usually located near the beam-injection point). Once the ions

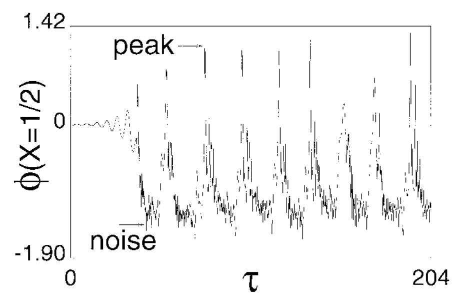



Fig. 5. The blocking oscillation when $\alpha/\pi = 3.26$ and $k_{De} = 59.5$. The peaks and blocks alternate with each other as time goes on with a low frequency ($\langle\omega_r\rangle = 0.32$). All the values marked in the figure are rescaled according to (4).

start to turn backward, the barriers grow higher by the ion accumulation near the barriers and the ions are trapped. Such unstable state lasts for some time and finally goes to the dc-current state [9], [11], where there is no oscillation and the potential difference between the bulk and the electrodes is so high that the assumption of immobility of dusts is not good.

### D. Blocking Oscillation

Between the dc-current and the chaotic oscillations, there is a narrow regime of blocking oscillations (the name is from [9] where the property of this oscillation is studied) as in Fig. 5. In the usual Pierce plasma diode, this mode is observed only when a thermal electron beam is used instead of a cold electron beam [9]. In fact, the blocking oscillation is the alternation between the dc-current state (no reflected particles) and an unstable state—the state where there are reflected and trapped ions. The noise in Fig. 5 is due to the trapped ions when the system is in the "unstable" state. The peak in Fig. 5 appears

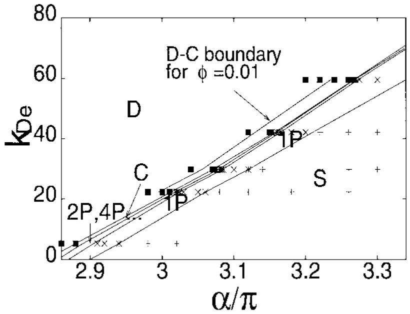



Fig. 6. The phase diagram ($k_{De}$ versus $\alpha/\pi$). The cross marks are the nonuniform equilibrium states ($S$), the $x$ marks are period one ($1P$), and the rectangles are the dc-current states ($D$). Since the period two or four ($2P$, $4P$) and the chaos ($C$) regimes are too narrow to mark any symbols, just the boundary lines are denoted.

when most of the trapped particles are drained out and the system is in the dc-current state. Therefore it can be thought that there is some mechanism of stabilizing the ion beam by reducing the potential barriers. In our system, it seems that the electron Debye shielding has an effect of stabilizing the ion beam. It is verified by the fact that the equilibrium state exists in the broader parameter regimes in our system than in the Pierce plasma diode using just a cold electron beam.

### *E. Phase Diagram*

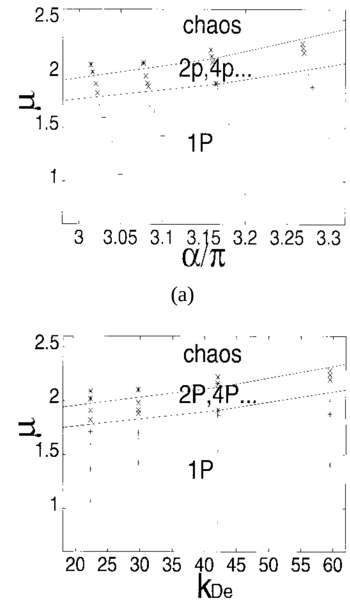
It is seen from (1)–(3) that the main control parameters are $\alpha$ and $k_{De}$ which are the combinations of many individual parameters $n_{i0}$, $v_{i0}$, $L$, $T_e$, $Zn_{d0}/n_{i0}$. Fig. 6 is the $k_{De}$ versus $\alpha/\pi$ phase diagram. The chaos ($C$) or the periodic oscillation ($1P$, $2P$, $\ldots$) regimes are so narrow that it is difficult to distinguish them in Fig. 6. The dc-current ($D$) and the equilibrium ($S$) regimes cover the large area of the phase diagram. The blocking oscillation ($B$) is not marked here since it is a very rare case. It is easily seen that the period-doubling "$S \rightarrow 1P \rightarrow 2P \rightarrow \cdots \rightarrow C \rightarrow (B) \rightarrow D$" will occur if the $\alpha$ decreases or the $k_{De}$ increases. The dust charge effect is reflected in the $k_{De} = L/\lambda_e = L\sqrt{Fe^2 n_i/\epsilon_0 T_e}$, where $F = 1 - Zn_{d0}/n_{i0}$. When the dust charge $Ze$ or the density $n_{d0}$ have large values, the $k_{De}$ becomes small which means the system can be more stable.

The $D$-$C$ boundary is shifted to the left when there is an external voltage. It is noted that as $k_{De}$ goes to zero, the $\alpha$-parameter regime of oscillation shifted to that of Pierce plasma diode (compare Figs. 1 and 6 in [10]).

## IV. Classification of Nonlinear Oscillations

The above phase diagram is just two-dimensional so its structure is simple, but if we considered another parameter effects such as dc or ac driving voltages, the phase diagram would have a three- or four-dimensional structure which may be very complex. The various phases can be simply classified in view of some single diagnostic parameter. This parameter is defined as follows:

$$\mu = \frac{L_s}{L_{slip}} = \frac{\pi v_{i0}}{\omega_{pi}} \left[ \frac{2\pi v_\phi (v_{i0} - v_\phi)}{\omega_b v_{i0}} \right]^{-1} = \frac{\omega_b/\omega_r}{2(1 - v_\phi/v_{i0})} \tag{8}$$




(a)

(b)

Fig. 7. The system states represented in diagrams of $\mu$ versus input parameters. The cross marks are $1P$, the $x$ marks are $2P$, $4P$, $\ldots$, and the asterisks represent the region of chaos.

where $v_\phi$ is the phase velocity of the wave inside the diode. The bounce frequency $\omega_b$ is defined as $\sqrt{ekE/m}$ with wave number $k$ and maximum electric field $E$ repulsing the beam at the beam-injection point ($X = 0$). The details on $L_s$ and $L_{slip}$ are in [10], [11], and [15]–[17].

It is seen in Fig. 7 that the phases of the system are dominantly dependent on $\mu$ rather than the input parameters $\alpha$ or $k_{De}$. This implies that the system state can be well classified by $\mu$ regardless of the input parameters.

## V. Conclusion

We studied the model of an ion-beam going through the dust and the Boltzmann electrons in the bounded system. A linear dispersion relation was derived for the perturbation about a uniform equilibrium. It was shown that there's no stable uniform equilibrium at least in $3 \leq \alpha/\pi \leq 7$ for large $k_{De}$ (in this paper we plotted $k_{De} = 59.5$ case). This analytic result was verified by many simulations for $3 \leq \alpha/\pi \leq 4$ and $20 < k_{De} < 60$. Instead of uniform equilibrium, there is a broad regime of nonuniform equilibrium in the $\alpha - k_{De}$ phase-diagram, which is contrasted with the usual Pierce diode [10], [11] where a uniform equilibrium is possible. There are period-doubling cascades between the dc-current mode and the equilibrium mode. We plotted the phase diagram of $k_{De}$ versus $\alpha/\pi$. It is shown that the system states are greatly influenced by the dusts and electrons. There appears a blocking oscillation mode which is thought to be possible due to the electron Debye-shielding over ions in the beam. Therefore this mode could not be found in the research of [10] and [11] where a cold electron beam was used.

All the oscillations described in this paper have ion time-scale since the ion-beam oscillates interacting with the dusts and the electrons, while the oscillations in a usual Pierce-diode have electron time-scale as in [9]–[11].

A diagnostic parameter was defined similarly as in [10] and [11] to classify the various phases. It has been shown that the phases of many beam-systems can be classified well in terms of this parameter [10], [11], [15]–[18] which is also suitable for our system, too.

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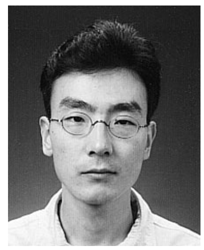

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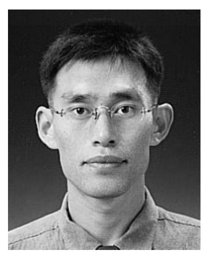

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**Jae Koo Lee**, for a photograph and biography, see this issue, p. 1371.