Technical Notes

Off-Peak Saturation Effects of Beam–Plasma Instability

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Abstract—**The saturation level, simulated by both conventional particle and hybrid simulations, verifies a theoretical estimation that the highest saturation level corresponds not to the fastest growing linear mode, but to the off-peak (in the linear growth rate curve) parameter.**

I. Introduction

Beam–plasma-type instabilities which are simulated using particle codes require a very large number of simulation particles for the plasma part, which is usually 100 to 1000 times denser than the beam part, in order that the noise from the plasma particles does not obscure the instability. We employ a hybrid code [1] using particles for simulation of the weak beam and a linearized fluid for the plasma. The linearization needs to be monitored but it is generally justifiable, since the plasma component stays linear in most weak beam–strong plasma interactions. Another similar scheme is in [2].

The linearized plasma like that of O'Neil *et al.* [3] is noiseless, allowing very low-level effects to be followed. We have used this code in this manner, and also to obtain saturation behavior where the plasma remains linear. We found that the peak amplitude (saturation) of electrostatic field energy for essentially single-mode excitation fell off very rapidly for k, away from that for maximum linear growth $\gamma_{\max}$ and more rapidly than did $\gamma/\gamma_{\max}$. This is predicted by single-mode trapping theory and verified by using both the particle code ES1 [4] and the hybrid code ES1 + EFL [1].

II. Theoretical Estimation

The single-wave trapping theory of Drummond *et al.* [5] provides a rough estimation of the saturation level of the electrostatic field energy relative to the initial beam kinetic energy for a weak cold beam–cold plasma instability with no magnetic field. The saturation level is defined as

$$\eta \equiv \frac{\frac{1}{4}\epsilon_o E^2}{\frac{1}{2} m_b n_b v_b^2} \tag{1}$$

where the perturbed electric field E is taken at the first peak. From [5], it is shown that

$$\eta \approx \frac{2\Delta v}{v_b} \tag{2}$$

where Δv is the difference between beam and phase velocities, and v_b is the beam velocity. If the most unstable mode (i.e., the fastest growing mode, $\gamma = \gamma_{\max}$, when the spectrum is dense and continuous, roughly at $kv_b/w_{pp} \simeq 1$, where w_{pp} is the plasma frequency of the plasma component) is allowed to grow, then the wave saturation is dominated by this fastest growing mode. A single-wave structure exists at least up to the first peak or saturation. In this single wave $\gamma = \gamma_{\max}$ case, (2) takes the more useful form, as in [5]:

$$\eta = \left(\frac{R}{2}\right)^{1/3} \tag{3}$$

where

$$R \equiv \frac{n_b}{n_p} = \left(\frac{w_{pb}}{w_{pp}}\right)^2 \tag{4}$$

n_b, n_p are beam and plasma densities, respectively, and w_{pb} is the plasma frequency of the beam.

However, if the system does not allow the excitation of the most unstable mode $\gamma_{\max}$ of the dense and continuous spectrum case, allowing only a single wave at $kv_b/w_{pp} \neq 1$, where $\gamma < \gamma_{\max}$, then the estimation formula (3) should be modified. This is because in deriving (3) it was assumed that the phase was that of the most unstable mode. This study was initially motivated by a computer simulation using $R = 0.001$ and only a single wave at $kv_b/w_{pp} = 0.8$, with $\gamma < \gamma_{\max}$, yielding $\eta = 0.01$, which is much less than $\eta = 0.079$ predicted by (3). We estimate η for a wide range of kv_b/w_{pp}, using the single-wave nonlinear trapping argument of [5]. Equation (2) will still be used to relate η and Δv. This relation needs modification when the plasma is not cold. We then compare the estimation with simulation results both by the particle code (ES1) [4] and the hybrid code (ES1 + EFL) [1]. In simulations, we fixed the weak beam strength at $R = 0.001$ and observed the saturation level η and the maximum growth rate γ/w_{pp} by changing the parameter kv_b/w_{pp}. We varied v_b; changing any other parameter (k or w_{pp}) yielded similar results in those cases that were checked. The dielectric function of the cold beam–cold plasma system is

$$\epsilon(k, w) = 1 - \frac{w_{pp}^2}{w^2} - \frac{w_{pb}^2}{(w - kv_b)^2} \equiv \epsilon_p(w) - \frac{w_{pb}^2}{(w - kv_b)^2}. \tag{5}$$

Let $f \equiv (kv_b/w_{pp})$ and $S \equiv w - kv_b$, where $|S| \ll 1$. Taylor expanding $\epsilon_p(w)$ around $w = kv_b$, we obtain (cf. Briggs [6]):

$$\begin{aligned}\epsilon_p(w) &= (\epsilon_p)_{w=kv_b} + S\left(\frac{\partial \epsilon_p}{\partial w}\right)_{w=kv_b} + O(S^2)\\ &= \left(1 - \frac{1}{f^2}\right) + \frac{2S}{f^3 w_{pp}} + O(S^2).\end{aligned} \tag{6}$$

Also, the dispersion relation from (5) is

$$\epsilon_p(w) = \frac{w_{pb}^2}{S^2}.$$

Hence

$$R = \frac{w_{pb}^2}{w_{pp}^2} = \left(1 - \frac{1}{f^2}\right)\frac{S^2}{w_{pp}^2} + \frac{2S^3}{f^3 w_{pp}^3} + O(S^4)$$

or

$$-\frac{2}{f^3}\chi^3 + \left(1 - \frac{1}{f^2}\right)\chi^2 \approx R \tag{7}$$

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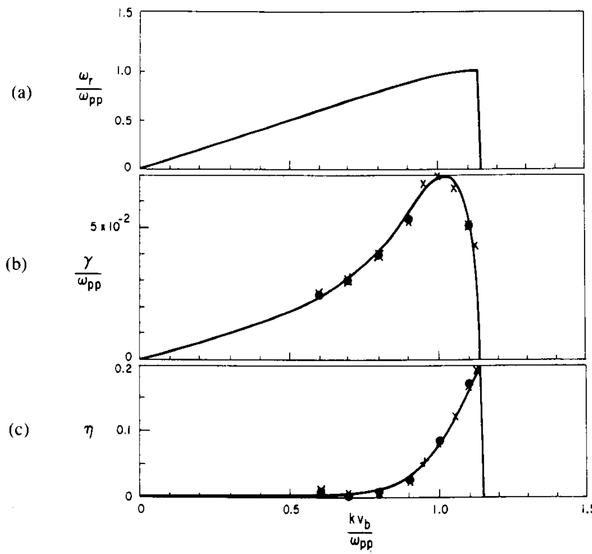

Fig. 1. Beam–plasma instability for $R = 0.001$: (a) Real frequencies of growing modes (the purely real frequencies of stable modes are not plotted); (b) growth rates; and (c) saturation levels. Curves are the theoretical predictions of equations (7)–(10). Single-mode simulation results are marked "○" (particle code) and "×" (hybrid code); real frequencies (a) also agree well with the theory (not shown).

where

$$\chi \equiv -\frac{S}{w_{pp}}.$$

When higher accuracy is necessary, the original quartic equation, (5), may be solved directly. Since S is a small parameter, we take (7) as our first approximation to the beam–plasma dispersion relation. Further, we obtain:

$$\text{Im}\,(\chi) = \text{Im}\,(w)/w_{pp} \equiv \gamma/w_{pp} \tag{8}$$

$$\text{Re}\,(\chi) = \frac{kv_b}{w_{pp}} - \frac{w_r}{w_{pp}} = f - \frac{w_r}{w_{pp}} \tag{9}$$

$$\eta = \frac{2\Delta v}{v_b} = \frac{2\,\text{Re}\,(\chi)}{f}. \tag{10}$$

The numerical solutions of (7)–(10) are plotted in Fig. 1. The saturation level (cf. Fig. 1(c)) increases monotonically, even past the maximum growth rate point $kv_b/w_{pp} = 1$. This is readily justified by the fact that the saturation level is directly proportional to the velocity slip (cf. (2)), namely, the difference between the phase velocity of the growing mode and beam velocity (cf. Fig. 1(a)).

III. Simulational Verification

A. Particle Simulation (ES1)

The input parameters used are quite modest: $n_b = 256$, $n_p = 128$, $w_{pp}\Delta t = 0.2$, $w_{pb}\Delta t \approx 6.3 \times 10^{-3}$, $L/\Delta x = 32$, $\tau_p/\tau_t \leq 1$ ($\tau_p = 2\pi/w_{pp}$, $\tau_t = L/v_b$). Both momentum conserving and energy conserving schemes are used as the charge and force weighting schemes. The excitation of modes is through density (i.e., position) perturbation. All wave numbers are excited initially with comparable amplitudes. The range from the starting wave energy to the saturation level is about 10^6 to 10^8. The results in Fig. 1 show very good agreement with those of our approximation formulas, (7)–(10). Saturation levels for all of the runs in which the most unstable mode may exist compare roughly with those calculated for the most unstable mode.

B. Hybrid Simulation (ES1 + EFL)

The input parameters for these cases are the same as in the corresponding particle simulations. Since this code uses linearized Eulerian fluid equations for the plasma component, the results are expected to be good when the linearity assumption holds; that is, when the ratio of the perturbed plasma velocity v_1 to the wave phase velocity is small compared with the unity. This was verified for most of the hybrid runs; for example, $(kv_1/w)_{max}$ was 0.57×10^{-2} for $k_{min}(v_b/w_{pp}) = 0.7$, and 1.9×10^{-2} for $k_{min}(v_b/w_{pp}) = 1.05$ (the linear assumption holds better for the former case).

Unlike the particle code, the hybrid simulation does not generate the nonlinear harmonics from the plasma component as waves grow in time; the nonlinear harmonics from the beam component are the same as in the wholly particle simulation. The overall difference between the particle and hybrid codes is small as long as the relative beam density is small, as in our case. The number of modes allowed yielded important differences. As an example, for $k_{min}(v_b/w_{pp}) = 0.7$, we obtained $\gamma/w_{pp} = 2.7 \times 10^{-2}$, $\eta = 2.2 \times 10^{-3}$ when only the fundamental mode was allowed and initially excited by a sinusoidal density modulation. This should be compared with $\gamma/w_{pp} = 3.1 \times 10^{-2}$, $\eta = 7.1 \times 10^{-3}$ (about three times larger) for $k_{min}(v_b/w_{pp}) = 0.7$, when all the modes are allowed during the nonlinear evolution of the modes (only the latter cases, i.e., all-modes-excitation, are plotted in Fig. 1). The amplitude of the initial mode energy is also an important factor for the study of the wave saturation. This level should be sufficiently low, but not too low as in laboratory experiments; otherwise, the simulation saturation levels will be appreciably affected. When the initial mode energy is too high, approaching the eventual saturation level, the mode evolution is not governed by the mode of interest because of the insufficient time for the instability development. When the initial mode energy is too low, below the omnipresent computer noise level, the simulation is dominated by nonphysical modes, masking the development of the physical mode of our interest. Thus the starting level of the mode of interest has to be determined between these two extremes, optimizing the computer time required to reach the saturation. This effect was observed in many of our runs listed here and in relativistic beam–plasma simulations [7].

IV. Conclusion

For beam–plasma instability in the absence of a magnetic field, hybrid simulations produce almost identical results to particle simulations (both of which agree with linear and nonlinear analyses), but with much reduced computing cost and noise level. With the verification by simulations, it is shown that the saturation level is very strongly influenced by the discrete wave number spectrum, sometimes giving different, by an order of magnitude, results by choosing slightly different off-peak (in growth rate curve) parameters, which must be considered in most simulations (whether hybrid or particle or whether magnetized or unmagnetized).

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