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Section A

1-D free-electron laser model without the slowly-varying approximation

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Abstract

A free-electron laser amplifier in the strong pump regime is studied without the slowly-varying envelope approximation (SVEA). A one-dimensional time-dependent code is used for numerical simulation of the evolution of the electron energy, the synchrotron phase of the electrons and the electric field of the laser. Electron-laser-facility-like parameters are used for the strong pump regime. Since the cooperation length is much smaller than the electron beam length, a steady-state solution is found to exist. Comparisons are made with the earlier results with the SVEA and the difference turns out to be negligible. It can be concluded that the SVEA can be applied to a wider class of problems than it sets out to be appropriate for.

1. Introduction

In this work, we study the temporal and spatial behavior of the free-electron laser (FEL) in the amplifier configuration without the so-called slowly-varying envelope approximation (SVEA).

An FEL is a device that generates coherent radiation out of incoherent radiation emitted by electrons wiggling under a periodically-varying external magnetic field (magnetic undulator) through the bunching of the electron beam by the radiation field [1]. A dimensional model of the FEL has been proven successful in reproducing many of the experimentally observed phenomena – for example, super-radiance [2–4] and super-radiant spikes [5–7]. It also predicts chaotic behaviors — transition from limit-cycle oscillations to chaos, quasi-periodicity and intermittency [8,9]. In previous works, the SVEA was applied to both analytical and the numerical studies [9,10]. The SVEA is valid when both the laser field amplitude and the laser phase change over length and time scales that are much longer than the laser wavelength and the laser frequency, respectively. As is pointed out in earlier works that used the SVEA, the results seem to indicate that the SVEA may not be an appropriate approximation since the laser amplitude and the phase change so rapidly as to violate the criterion for the SVEA. This observation motivates the work without the SVEA [11].

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2. Model

The FEL model consists of a set of three equations: One, for the evolution of the electron energy through the interaction between the electron and the electric field of the laser; another, for the synchrotron (or ponderomotive) phase of the electron due to the changing axial velocity of the electron; the past, the electromagnetic wave equation with a current source arising from the electron motion. Space-charge effects are neglected because the plasma-oscillation frequency is much smaller than the laser frequency. The advantage of the SVEA is that it reduces the wave equation to a first-order differential equation instead of second order which is easier to handle. The equations are

$$\frac{\mathrm{d}}{\mathrm{d}t}\gamma = -\frac{a_\mathrm{w}(z)E_y(z,t)}{\gamma c}, \tag{1}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\psi = c[(k_\mathrm{w}+k)\beta_z - k], \tag{2}$$

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}\right)f^{+} = -F_\mathrm{b}\left(\frac{\omega_\mathrm{p}^2}{\gamma c}\right)a_\mathrm{w}(z), \tag{3}$$

and

$$\left(\frac{\partial}{\partial t} - c\frac{\partial}{\partial z}\right)f^{-} = -F_\mathrm{b}\left(\frac{\omega_\mathrm{p}^2}{\gamma c}\right)a_\mathrm{w}(z). \tag{4}$$

In Eqs. (1)–(4), γ is the relativistic factor, z is the coordinate along the wiggler axis, c is the speed of light, $a_\mathrm{w}(z) = a_{\mathrm{w}_0} \sin k_\mathrm{w} z$ is the wiggler vector potential in the

y direction, E_y is the electric field of the radiation along the y direction, $\psi \doteq (k_w + k)z - \omega t$ is the synchrotron phase, k_w and k are the wave numbers of the wiggler and the radiation $k = \omega/c$, β_x is the ratio of the electron velocity to the speed of light, F_0 is the filling factor simulating the three dimensional effect [10], and ω_p the frequency of the plasma oscillation. In addition, we define

$$\frac{d}{dt} \doteq \frac{\partial}{\partial t} + v_z \frac{\partial}{\partial z}, \tag{5}$$

and

$$f^{\pm} \doteq B_x \mp E_y/c, \tag{6}$$

where B_x is the magnetic field of the radiation in the x direction. Here, a_w, E_y and B_x are normalized by e/mc. In the SVEA, $f^- \approx 0$ and $f^+ \approx -(2/c)E_y$ and one ends up with three equations (1)–(3).

In order to obtain the amplitude and the phase of the electric field it is convenient to introduce the complex fields $\tilde{f}^{\pm}$ such that $f^{\pm} = \mathscr{T}\tilde{f}^{\pm}$. By separating the complex field into two parts $\tilde{f}^{\pm} \doteq \tilde{g}^{\pm} e^{i(kz - \omega t)}$ one can find the amplitude and the phase from $\tilde{g}^{\pm}$ by averaging over the electrons. In numerically integrating Eqs. (1)–(4) we do the following [12]: As electrons advance by one step size Δz along the axis, g^+ slips ahead while g^- slips behind, both by the length $(\lambda/\lambda_\omega)\Delta z$ that is the same as the mesh size with which the electron beam is finite-differenced. At each mesh point a finite number of electrons is distributed. The input for the $f^{\pm}$ is $f^+ = -2ka_0 \cos(kz - \omega t + \phi)$ and $f^- = 0$. As Tsui points out [10], for strong pumping a_{w0} larger than 1 it is appropriate to use $\gamma^2 = (1 + a_w^2)/(1 - \beta_z^2)$ to express β_z in terms of γ.

The input parameters for the simulations are [10]: All the electrons have equal energy with $\gamma = 6.8$ (3.1 MeV), $a_{w0} = 3$ ($B_w = 3.2$ kG), $\lambda = 0.87$ cm, $\lambda_w = 9.8$ cm, wiggler length $L_w = 30\lambda_w$, beam current 850 A, beam cross sectional area 0.64 cm^2, the plasma frequency $\omega_p = 3 \times 10^{10}$ rad/s, the filling factor $F_b = 0.2$ and the input microwave power 50 kW (i.e., $a_0 = 1.47 \times 10^{-3}$). It is found that ten electrons at each mesh point is sufficient for our purpose.

3. Results

The electron energy γ and the gain $G = 10 \log(E_{y0}^2/E_0^2)$ in two typical cases with and without the SVEA are plotted in Figs. 1 and 2. Here, E_{y0} is the amplitude of the electric field and E_0 is the input amplitude. In Fig. 1, the beam size $L_b = 6\lambda$ and, in Fig. 2, $L_b = 30\lambda$. In both cases, the results are taken at ten periods into the wiggler. Hence, one can conclude that although the SVEA does introduce detailed differences in the results obtained without the SVEA, the results with the SVEA do behave the same on the average as the non-SVEA results. It is, thus, expected that with the SVEA the global quantities such as the total power and the average efficiency are not far off from the values without the SVEA. It is yet to be worked out whether the same conclusion can be reached in the case of an oscillator configuration because the backward part of the field has more opportunities to interact with the electrons inside the resonator. Work in this direction is in progress and will be presented elsewhere.

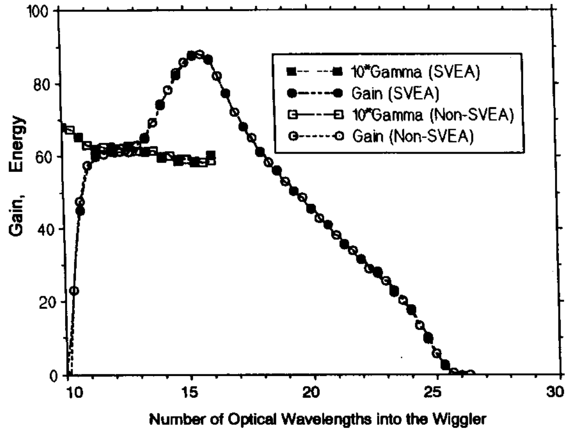

Fig. 1. Gain profile and electron energy at ten wiggler periods into the wiggler: Beam size $l_b = 6\lambda$; open marks for the results without the SVEA and the filled marks for the case with the SVEA; circles for the gain and the squares for 10γ; the unit of the horizontal axis is the radiation wavelength λ.

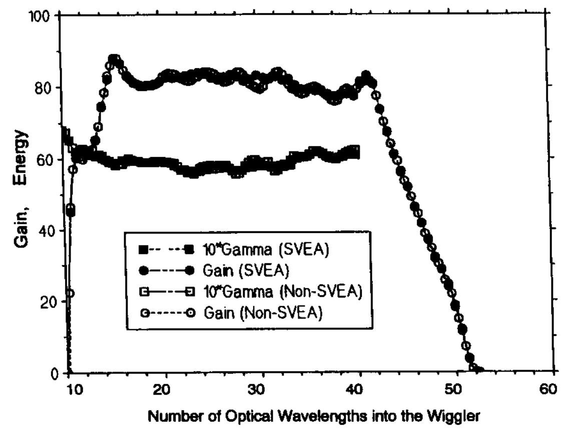

Fig. 2. The same caption as in Fig. 1 except that the beam size $l_b = 30\lambda$.

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