Theory of Wave Propagation Along Corrugated Waveguide Filled with Plasmas Immersed in an Axial Magnetic Field

Liu Shenggang, J. K. Lee, Song Liqun, Yan Yang, and Zhu Dajun

***Abstract*—Analytical theory of wave propagation along a corrugated waveguide filled with plasmas immersed in a finite axial magnetic field is presented in this paper. Preliminary numerical calculations are also done, showing that the intensity of magnetic field has considerable influence on the dispersive characteristics of wave propagation and the properties of such devices.**

I. Introduction

Cylindrical corrugated waveguide has been widely used and studied [1]–[12]. It has been found that in relativistic backward wave oscillators (BWO's) with rippled wall, a great efficiency increase may be obtained when the waveguide is filled with plasma, and the dispersive characteristics for this case are also drastically changed [4]–[8]. The published theoretical studies, however, only dealt with the cases where the magnetic field inclines to zero or infinity. In practice, a finite axial magnetic field is often used for confining the electron beam [4]–[14]. A more rigorous theory which covers all the cases, therefore, should be worked out.

This paper gives detailed analytical theory of wave propagation along a cylindrical corrugated waveguide filled with plasmas when the finite magnetic field is taken into account. Floquet's expansion of field components, eigenvalues, and dispersion equations are derived to investigate all cases of corrugated structure. Complex wave power transmission and coupling coefficients in some frequently used cases are discussed in detail. Also, numerical calculations have been conducted. From the dispersion curves, it is shown clearly that effects of the finite magnetic field considerably influence dispersion characteristics of wave propagation along corrugated plasma waveguides and thus the properties of such devices.

II. Basic Equations

Assuming that collision effects have been neglected, the electric permittivity tensor of magnetized plasmas is

$$\bar{\varepsilon} = \begin{vmatrix} \varepsilon_1 & -\varepsilon_2 & 0 \\ \varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{vmatrix} \tag{1}$$

Manuscript received August 15, 1995; revised March 25, 1996. This work was supported by the Chinese National Science Foundation.

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Publisher Item Identifier S 0093-3813(96)05661-5.

where

$$\varepsilon_1 = 1 - \xi^2/(1-\tau^2); \quad \varepsilon_2 = j\tau\xi^2/(1-\tau^2)$$
$$\varepsilon_3 = 1 - \xi^2 \tag{2}$$

with

$$\xi^2 = \omega_p^2/\omega^2; \quad \tau^2 = \omega_c^2/\omega^2 \tag{3}$$

where $\omega_p^2 = (e^2 n_0)/(\varepsilon_0 m_0)$ is the plasma frequency and $\omega_c = (|e|B_0)/m_0$ is the cyclotron frequency.

For a uniform waveguide completely filled with plasmas, the longitudinal field components may be written as follows [19]:

$$E_z = A_1 J_m(p_1 r) + A_2 J_m(p_2 r) \tag{4}$$
$$H_z = A_1 h_1 J_m(p_1 r) + A_2 h_2 J_m(p_2 r) \tag{5}$$

where $p_{1,2}^2$ are the eigenvalues and $J_m(p_{1,2}r)$ are m-order Bessel functions with complex arguments. The transverse field components can be expressed by E_z and H_z. Detailed derivations and expressions can be found in [19].

Since the corrugated waveguide is a periodic structure, the field components should be expanded into space harmonics according to Floquet's theorem

$$E_z = \sum_n [A_{1,n} J_m(p_{1,n} r) + A_{2,n} J_m(p_{2,n} r)] e^{-jk_{II,n} z} \tag{6}$$

$$H_z = \sum_n [A_{1,n} h_{1,n} J_m(p_{1,n} r) + A_{2,n} h_{2,n} J_m(p_{2,n} r)] e^{-jk_{II,n} z}. \tag{7}$$

Here we have neglected the attenuation of wave propagation, and assume that: $\gamma = jk_{//}$, so we have

$$k_{//,n} = k_{//} + 2\pi n/L \tag{8}$$

L is the space period and n is the harmonical order. The eigenvalues $(p_{1,2})_n^2$ may be found as

$$(p_{1,2})_n^2 = \frac{1}{2\varepsilon_1}[-k_{II,n}^2(\varepsilon_1+\varepsilon_3) + k^2(\varepsilon_1\varepsilon_3 + \varepsilon_1^2 + \varepsilon_2^2)] \pm \frac{1}{2\varepsilon_1}\{[-k_{II,n}^2(\varepsilon_3-\varepsilon_1) + k^2(\varepsilon_1\varepsilon_3 - \varepsilon_1^2 - \varepsilon_2^2)]^2 - 4k^2 k_{II,n}^2 \varepsilon_2^2 \varepsilon_3\}^{1/2} \tag{9}$$

and

$$(h_{1,2})_n = \left[(-k_{II,n}^2 + k^2\varepsilon_1)\frac{\varepsilon_3}{\varepsilon_1} - (p_{1,2})_n^2\right] \Big/ \left(-\omega\mu_0 k_{II,n}\frac{\varepsilon_2}{\varepsilon_1}\right). \tag{10}$$

In (6) and (7), A_{1n} and A_{2n} are undetermined coefficients. We can also get the expressions of transverse field components in terms of E_z and H_z by using the same derivation procedure as [19]. Note that at present E_z, H_z and then $E_r, E_\theta, H_r, H_\theta$ have been derived with no approximations. Thus, they are vigorous and general equations covering all cases for magnetized plasma corrugated waveguides.

III. Dispersion Equations

The boundary conditions may be written as

$$r = a + h\cos k_0 z \quad E_t = 0 \quad H_n = 0. \tag{11}$$

Here, we have assumed that the corrugated waveguide has sinusoidal wall, h is the amplitude of the ripple, a is the average radius, and $k_0 = 2\pi/L$. E_t, H_n are the tangential and normal components to the wall surface, respectively. Equation (11) may be specified as

$$E_z - E_r k_0 h \sin k_0 z = 0 \tag{12}$$

$$H_z k_0 h \sin k_0 z + H_r = 0. \tag{13}$$

Substituting (6), (7), E_r, and H_r into (12) and (13), and taking the average in one period, then we get the general dispersion equation

$$\begin{bmatrix} (F_{m,1})_{n,s} & (F_{m,2})_{n,s} \\ (G_{m,1})_{n,s} & (G_{m,2})_{n,s} \end{bmatrix} = 0. \tag{14}$$

The elements of the above equation can be seen in Appendix I.

Now we can discuss some special cases:

1) For $m = 0$, which represents symmetrical modes, we have

$$\begin{bmatrix} (F_{0,1})_{n,s} & (F_{0,2})_{n,s} \\ (G_{0,1})_{n,s} & (G_{0,2})_{n,s} \end{bmatrix} = 0. \tag{15}$$

Putting $m = 0$ in (39)–(42), we get the elements in (15): $(F_{0,1})_{n,s}, (F_{0,2})_{n,s}, (G_{0,1})_{n,s}$, and $(G_{0,2})_{n,s}$.

2) When $\omega_c \to \infty (B_{0z} \to \infty)$ and $\omega_c \to 0 (B_{z0} \to 0)$, we get $k_g \to 0$ and then decoupled wave equations of E_z and H_z for both cases. Thus TE modes and TM modes are independent from each other.

For TM modes, the dispersion equation (14) reduces to

$$|(F_{m,1})_{n,s}| = 0, \quad \text{and} \quad |(G_{m,2})_{n,s}| = 0 \text{ for TE modes.} \tag{16}$$

For symmetric modes, (16) may be further simplified as

$$|(F_{0,1})_{n,s}| = 0 \quad \text{and} \quad |(G_{0,2})_{n,s}| = 0 \tag{17}$$

where

$$(F_{0,1})_{n,s} = \frac{1}{L}\int_{-(L/2)}^{L/2} dz \cdot \left[J_0(p_{1,n}r) + \frac{jk_{II,n}p_{1,n}J_1(p_{1,n}r)}{k^2\varepsilon_1 - k_{II,n}^2} k_0 h \sin k_0 z\right] \cdot e^{-j(n-s)k_0 z} \tag{18}$$

and

$$(G_{0,2})_{n,s} = \frac{1}{L}\int_{-(L/2)}^{L/2} dz \cdot \left[J_0(p_{2,n}r)k_0 h \sin k_0 z - \frac{jk_{II,n}K_n^2 p_{2,n}J_1(p_{2,n}r)}{k^2\varepsilon_1 - k_{II,n}^2}\right] \cdot e^{-j(n-s)k_0 z} \tag{19}$$

respectively. Equation (18) for $B_{0z} \to \infty$ is the result given in [3]–[5].

In (16)–(19), eigenvalues can be written as

$$p_{1,n}^2 = (-k_{II,n}^2 + k^2\varepsilon_1)\varepsilon_3/\varepsilon_1 \tag{20}$$

for TM_{0n} modes and

$$p_{2,n}^2 = -k_{II,n}^2 + k^2\varepsilon_1 \tag{21}$$

for TE_{0n} modes, where $\varepsilon_1 = 1, \varepsilon_3 = 1 - \omega_p^2/\omega^2$ for $B_{0z} \to \infty$ and $\varepsilon_1 = \varepsilon_3 = 1 - \omega_p^2/\omega^2$ for $B_{z0} \to 0$.

Thus, the general dispersion equation (14) may cover all special cases.

IV. Wave Power Transmission

Now we can calculate the wave power transimission

$$P = \int_0^{2\pi} d\theta \int_0^{R_c} dr(E_r H_\theta^* - E_\theta H_r^*). \tag{22}$$

Substituting the expressions of the transverse field components into (22), we obtain

$$P = \sum_n P_n = \sum_n \frac{2\pi A R_c}{D_n D_n^*} J_m^2(p_{1,n}R_c) J_m^2(p_{2,n}R_c) \cdot \left[\left(\frac{m}{R_c}\right)^2 T_{1n} + \left(\frac{m}{R_c}\right) T_{2n} + T_{3n} + \frac{J_m'(p_{1,n}R_c)}{J_m(p_{1,n}R_c)} T_{4n} + \frac{J_m'(p_{2,n}R_c)}{J_m(p_{2,n}R_c)} T_{5n} + \frac{J_m'^2(p_{1,n}R_c)}{J_m^2(p_{1,n}R_c)} T_{6n} + \frac{J_m'^2(p_{2,n}R_c)}{J_m^2(p_{2,n}R_c)} T_{7n}\right] \tag{23}$$

in which the coefficients from T_{1n} to T_{7n} are written in Appendix II.

For symmetric mode, $m = 0$, we have

$$(P)_{m=0} = \sum_n (P_n)_{m=0} \tag{24}$$

and

$$P_n(m=0) = \frac{2\pi A R_c}{D_n D_n^*} J_0^2(p_{1,n}R_c) J_0^2(p_{2,n}R_c) \cdot \left[T_{3n} - \frac{J_1(p_{1,n}R_c)}{J_0(p_{1,n}R_c)} T_{4n} - \frac{J_1(p_{2,n}R_c)}{J_0(p_{2,n}R_c)} T_{5n} + \frac{J_1^2(p_{1,n}R_c)}{J_0^2(p_{1,n}R_c)} T_{6n} + \frac{J_1^2(p_{2,n}R_c)}{J_0^2(p_{2,n}R_c)} T_{7n}\right]. \tag{25}$$

In obtaining the above equations, the formulas of integration and the recurring relations of Bessel functions are utilized.

The calculation of the coupling coefficient will begin with its definition

$$(K_{\text{coup}})_n = \left(\frac{1}{L}\int_{-(L/2)}^{L/2} |E_z|\, dz\right)^2 \Big/ 2P \tag{26}$$

where $|E_z|$ is

$$E_z = \sum_n |[A_{11n}J_m(p_{1,n}r) + A_{21n}J_m(p_{2,n}r)]e^{-jk_{//,n}z}|. \tag{27}$$

Substituting (23) and (27) into (26), we get

$$(K_{\text{coup}})_n = |A_{1,n}J_m(p_{1,n}r) + A_{2,n}J_m(p_{2,n}r)|^2 \cdot \left(\frac{1}{L}\sin\frac{k_{//}L}{2}\right)^2 \Big/ 2\sum_n P_n. \tag{28}$$

The coupling coefficient is very useful in the theory of relativistic microwave devices [2]–[10].

For $m = 0, n = \pm 1$, we have

$$(K_{\text{coup}})_{n=\pm 1} = \left(|A_{1,\pm 1}J_0(p_{1,\pm 1}r) + A_{2,\pm 1}J_0(p_{2,\pm 1}r)|^2 \cdot \left(\frac{1}{L}\sin\frac{k_{//}L}{2}\right)^2\right) \Big/ 2\left(\sum_n P_n\right)_{m=0}. \tag{29}$$

And

$$(K_{\text{coup}})_{n=\pm 1} = \left(|A_{1,\pm 1}J_1(p_{1,\pm 1}r) + A_{2,\pm 1}J_1(p_{2,\pm 1}r)|^2 \cdot \left(\frac{1}{L}\sin\frac{k_{//}L}{2}\right)^2\right) \Big/ 2\left(\sum_n P_n\right)_{m=1} \tag{30}$$

for $m = 1, n = \pm 1$.

Now we can discuss the two special cases of $\omega_c \to \infty (B_{0z} \to \infty)$ and $\omega_c \to 0 (B_{0z} \to 0)$.

For TM modes

$$P = 2\pi A R_c \sum_n \frac{1}{D_n D_n^*} J_m^2(p_1 R_c) \cdot \left\{\left(\frac{m}{R_c}\right)^2 T_1 + \left(\frac{m}{R_c}\right) T_2 + T_3 + \frac{J_m'^2(p_1 R_c)}{J_m^2(p_1 R_c)} T_6\right\}. \tag{31}$$

In (31), T_1, T_2, T_3, and T_6 are simplified as

$$T_1 = -j\omega\varepsilon_0\varepsilon_1 k_{//,n} k_n^2 k_n^{*2} \quad T_2 = 2j\omega\varepsilon_0\varepsilon_1 k_{//,n} k_n^2 k_n^{*2} \tag{32}$$

$$T_3 = \tfrac{1}{2}\omega\varepsilon_0\varepsilon_1 p_{1,n}^2 k_n^2 k_n^{*2}$$
$$T_6 = -\tfrac{1}{2}j\omega\varepsilon_0\varepsilon_1 k_{//,n} p_{1,n}^2 k_n^2 k_n^{*2}. \tag{33}$$

For TE modes

$$P = 2\pi A R_c \sum_n \frac{1}{D_n D_n^*} J_m^2(p_2 R_c) \cdot \left\{\left(\frac{m}{R_c}\right)^2 T_1 + \left(\frac{m}{R_c}\right) T_2 + T_3 + \frac{J_m'^2(p_2 R_c)}{J_m^2(p_2 R_c)} T_7\right\} \tag{34}$$

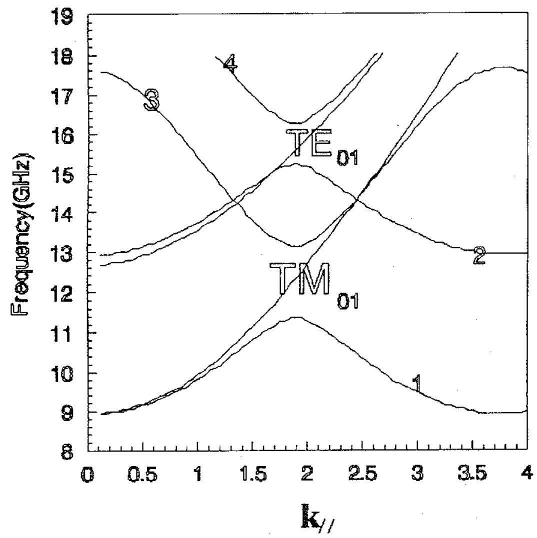

Fig. 1. Infinite magnetic field.

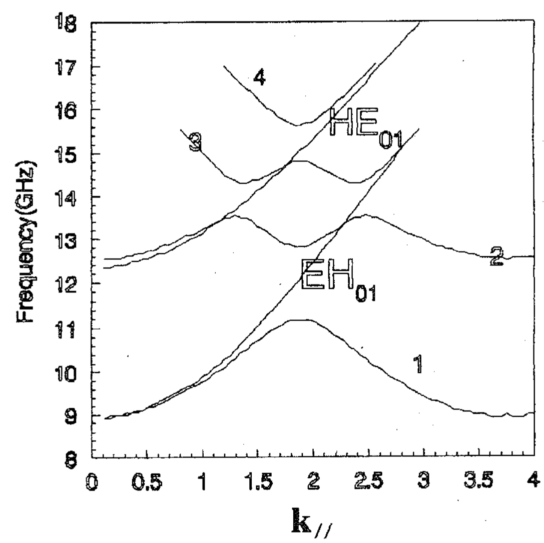

Fig. 2. 7078 G.

where T_1, T_2, T_3, and T_7 are

$$T_1 = -j\omega\mu_0 k_{//,n} k_n^2 k_n^{*2}$$
$$T_2 = j\omega k_{//,n} k_n^2 k_n^{*2}(2\varepsilon_0\varepsilon_1 + \mu_0) \tag{35}$$
$$T_3 = j\omega p_{2,n}^2 k_{//,n} k_n^2 k_n^{*2}(\tfrac{1}{2}\varepsilon_0\varepsilon_1 + \mu_0) \tag{36}$$
$$T_7 = \tfrac{1}{2}j\omega k_{//,n} p_{2,n}^2 k_n^2 k_n^{*2}(\varepsilon_0\varepsilon_1^* + \mu_0) \tag{37}$$

where $\varepsilon_1 = 1$ for $B_{0z} \to \infty$ and $\varepsilon_1 = 1 - \omega_p^2/\omega^2$ for $B_{z0} \to 0$.

So, for TM modes, we can calculate the coupling coefficient

$$(K_{\text{coup}})_m^{\text{TM}} = \left(|A_{m,n}J_m(p_{1,n}r)|^2 \left(\frac{1}{L}\sin\frac{k_{//}L}{2}\right)^2\right) \Big/ 2(P)^{\text{TM}}. \tag{38}$$

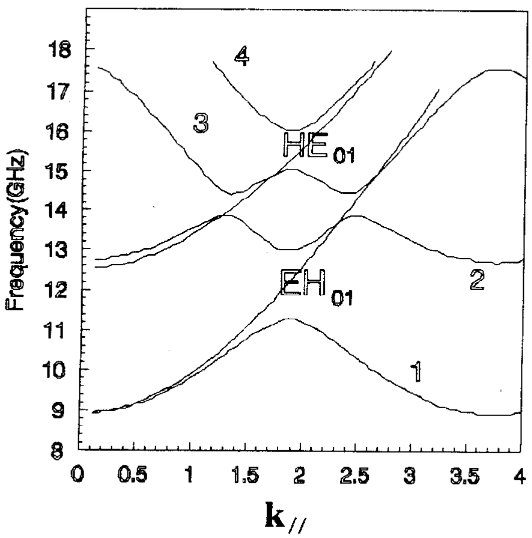

Fig. 3. 9437 G.

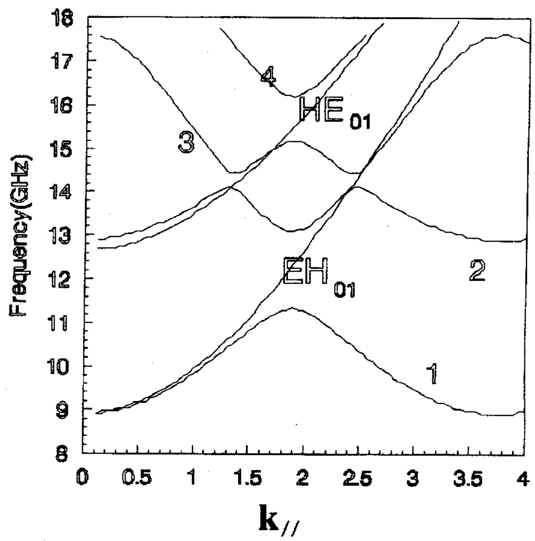

Fig. 4. 16514 G.

In (38), $(P)^{\mathrm{TM}}$ is given in (31)–(33). From the gerneral equation (28), we easily get the expressions for $(K_{\mathrm{coup}})^{\mathrm{TM}}$ for $m = 0$ and $m = 1$ for $n = \pm 1$.

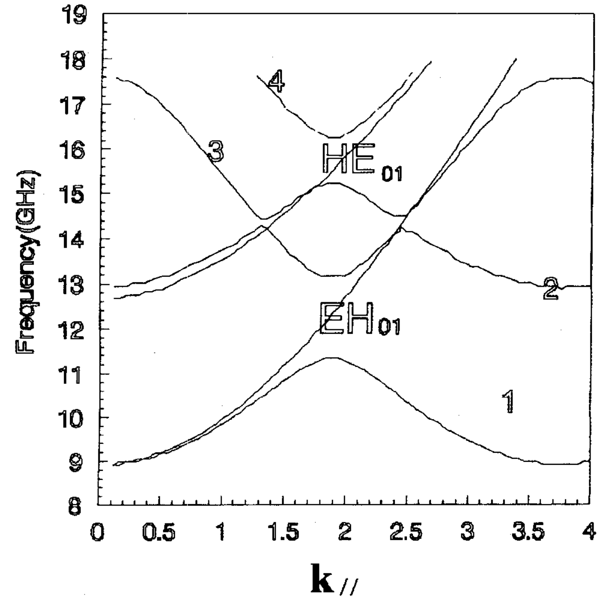

Fig. 5. 35388 G.

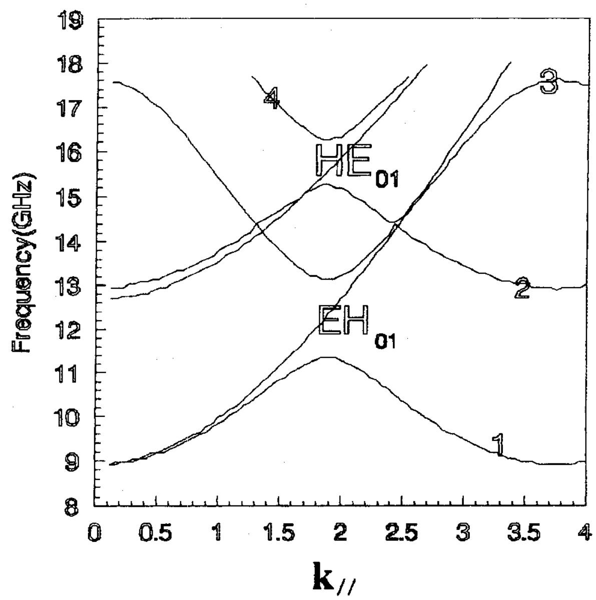

Fig. 6. 58980 G.

V. Numerical Calculation

In this section, numerical calculations of the dispersion equation for symmetric modes in corrugated waveguide filled with plasmas in finite magnetic field are given. The parameters employed in the calculations are

Space Period:	$L_p = 1.67$ (cm)
Ripple Amplitude:	$h = 0.145$ (cm)
Wave Guide Radius:	$R = 1.445$ (cm)
Plasma Density:	$N_b = 2.0 \times 10^{11}$ (cm^{-1}).

The 18×18 (the maximal space harmonic is four) determinant is used in the calculations. Six figures have been obtained. At first, it is verified that the same results are obtained as previous papers [18] for the infinite magnetic field illustrated in Fig. 1. Figs. 2–6 correspond to a finite magnetic field with magnetic field intensity 7078 G, 9437 G, 16514 G, 35388 G, 58980 G, respectively. In Fig. 1, curves 1 and 3 are space harmonics of TM_{01}. Curves 2 and 4 denote space harmonics of TE_{01}. However, for finite magnetic field, curves 1 and 3 represent EH_{01} and curves 2 and 4 represent HE_{01}, where the nomination system of plasma waveguides is involved [19]. It can be seen from Figs. 2–6 that the dispersion curves for every mode are expanded into space harmonics, as expected by the theory. It also shows that when $B_0 \to \infty$, the dispersion curves of the TM_{01} and TE_{01} modes cross each other at two points in each period and when the magnetic field B_0 decreases, these two curves are gradually seperated and become EH_{01} and HE_{01}, respectively. It can be concluded that the intensity of magnetic field B_0 gives quite strong influences on the dispersion characteristics of the wave propagation along corrugated plasma waveguide and thus the properties of such devices.

VI. Conclusion

This paper examines wave propagation along a plasma-filled corrugated waveguide immersed in a definite axial magnetic field. Based on the authors' previous papers, the vigorous and general field component expressions by using Floquet's expansion are gained. Complex dispersion equations covering all cases have been derived. Also, this paper discusses the expressions of wave power transmission and coupling coefficients in some special and useful cases. The equations obtained may be an effective way to solve problems about complex plasma corrugated structures. Preliminary numerical calculations are given, illustrating that the intensity of magnetic field greatly affects the dispersion characteristics of wave propagation along corrugated plasma waveguide and thus the properties of such devices.

Appendix I

The elements of the dispersion equation (14)

$$\begin{aligned}(F_{m,1})_{n,s} =& \frac{1}{L}\int_{-(L/2)}^{L/2} dz\Big\{J_m(p_{1,n}r) \\ &+ \frac{1}{D_n}\Big[jk_{II,n}K_n^2 p_{1,n}J_m^{(}p_{1,n}r) \\ &+ j\frac{m}{r}k_g^2 k_{II,n}J_m(p_{1,n}r) \\ &- \omega\mu_0 k_g^2 h_{1,n}p_{1,n}J_m'(p_{1,n}r) \\ &- \frac{m}{r}\omega\mu_0 K_n^2 h_{1,n}J_m(p_{1,n}r)\Big]k_0 h\sin k_0 z\Big\} \\ &\cdot e^{-j(n-s)k_0 z}\end{aligned} \tag{39}$$

$$\begin{aligned}(F_{m,2})_{n,s} =& \frac{1}{L}\int_{-(L/2)}^{L/2} dz\Big\{J_m(p_{2,n}r) \\ &+ \frac{1}{D_n}\Big[jk_{II,n}K_n^2 p_{2,n}J_m'(p_{2,n}r) \\ &+ j\frac{m}{r}k_g^2 k_{II,n}J_m(p_{2,n}r) \\ &- \omega\mu_0 k_g^2 h_{2,n}p_{2,n}J_m'(p_{2,n}r) \\ &- \frac{m}{r}\omega\mu_0 K_n^2 h_{2,n}J_m(p_{2,n}r)\Big]k_0 h\sin k_0 z\Big\} \\ &\cdot e^{-j(n-s)k_0 z}\end{aligned} \tag{40}$$

$$\begin{aligned}(G_{m,1})_{n,s} =& \frac{1}{L}\int_{-(L/2)}^{L/2} dz\; e^{-j(n-s)k_0 z} \\ &\cdot\Big\{h_{1,n}J_m(p_{1,n}r)k_0 h\sin k_0 z \\ &+ \frac{1}{D_n}\Big[-\omega\varepsilon_0 k_{II,n}^2\varepsilon_g p_{1,n}J_m'(p_{1,n}r) \\ &- \omega\varepsilon_0\frac{m}{r}(\varepsilon_1 K_n^2 - \varepsilon_g k_g^2)J_m(p_{1,n}r) \\ &- jk_{II,n}K_n^2 h_{1,n}p_{1,n}J_m'(p_{1,n}r) \\ &- jk_{II,n}k_g^2\frac{m}{r}h_{1,n}J_m(p_{1,n}r)\Big]\Big\}\end{aligned} \tag{41}$$

$$\begin{aligned}(G_{m,2})_{n,s} =& \frac{1}{L}\int_{-(L/2)}^{L/2} dz\; e^{-j(n-s)k_0 z} \\ &\cdot\Big\{h_{2,n}J_m(p_{2,n}r)k_0 h\sin k_0 z \\ &+ \frac{1}{D_n}\Big[-\omega\varepsilon_0 k_{II,n}^2\varepsilon_g p_{2,n}J_m'(p_{2,n}r) \\ &- \omega\varepsilon_0\frac{m}{r}(\varepsilon_1 K_n^2 - \varepsilon_g k_g^2)J_m(p_{2,n}r) \\ &- jk_{II,n}K_n^2 h_{2,n}p_{2,n}J_m'(p_{2,n}r) \\ &- jk_{II,n}k_g^2\frac{m}{r}h_{2,n}J_m(p_{2,n}r)\Big]\Big\}\end{aligned} \tag{42}$$

with

$$D_n = K_n^4 - k_g^4 \quad K_n^2 = -k_{II,n}^2 + k^2\varepsilon_1. \tag{43}$$

Appendix II

The coefficients in the expression of wave power transmission

$$\begin{aligned}T_{1,n} =& \frac{1}{2}[-k_{//,n}^2(h_{1,n}^* + h_{2,n}^*)(k_g^{*^2}K_n^2 + k_g^2 K_n^{*^2}) \\ &+ 2jk_{//,n}\varepsilon_0\omega(-\varepsilon_g^* k_{//,n}^2 k_g^2 + \varepsilon_g^* k_g^{*^2}K_n^2 - \varepsilon_1^* K_n^2 K_n^{*^2}) \\ &+ j\mu_0\omega k_{//,n}(k_g^2 k_g^{*^2} + K_n^2 K_n^{*^2}) \\ &\cdot(h_{1,n}^* h_{2,n} + h_{1,n}h_{2,n}^*) - k^2 k_g^2(\varepsilon_g^* k_g^{*^2} - \varepsilon_1^* K_n^{*^2}) \\ &\cdot(h_{1,n} + h_{2,n}) + \varepsilon_g^* k_{//,n}^2 k^2 K_n^2(h_{1,n} - h_{2,n})]\end{aligned} \tag{44}$$

$$\begin{aligned}T_{2,n} =& k_{//,n}^2(h_{1,n}^* + h_{2,n}^*)(k_g^{*^2}K_n^2 + k_g^2 K_n^{*^2}) \\ &- 2\varepsilon_0 k_{//,n}\omega(-\varepsilon_g^* k_{//,n}^2 k_g^2 + \varepsilon_g^* k_g^{*^2}K_n^2 - \varepsilon_1^* K_n^2 K_n^{*^2}) \\ &- j\mu_0\omega k_{//,n}(k_g^{*^2}K_n^2 + k_g^2 K_n^{*^2}) \\ &\cdot(h_{1,n}h_{1,n}^* + h_{2,n}h_{2,n}^*) + j\mu_0\omega k_{//,n} \\ &\cdot(k_g^2 k_g^{*^2} + K_n^2 K_n^{*^2} + k_g^{*^2}K_n^2 + k_g^2 K_n^{*^2}) \\ &\cdot(h_{1,n}h_{2,n}^* + h_{1,n}^* h_{2,n}) + k^2(\varepsilon_g^* k_g^2 k_g^{*^2} - \varepsilon_g^* k_{//,n}^2 K_n^2 \\ &- \varepsilon_1^* k_g^2 K_n^{*^2})(h_{1,n} + h_{2,n})\end{aligned} \tag{45}$$

$$\begin{aligned}T_{3,n} =& -\frac{1}{2}[-k_{//,n}^2(h_{1,n}^* p_{2,n}^2 + h_{2,n}^* p_{1,n}^2) \\ &\cdot(k_g^{*^2}K_n^2 + k_g^2 K_n^{*^2}) + j\varepsilon_0\omega k_{//,n} \\ &\cdot(-\varepsilon_g^* k_{//,n}^2 k_g^2 + \varepsilon_g^* k_g^{*^2}K_n^2 - \varepsilon_1^* K_n^2 K_n^{*^2}) \\ &\cdot(p_{1,n}^2 + p_{2,n}^2) - j\mu_0\omega k_{//,n}(k_g^{*^2}k_g^2 + K_n^2 K_n^{*^2}) \\ &\cdot(h_{1,n}h_{2,n}^* p_{1,n}^2 + h_{1,n}^* h_{2,n}p_{2,n}^2) \\ &- k^2(h_{1,n}p_{1,n}^2 + h_{2,n}p_{2,n}^2) \\ &\cdot(\varepsilon_g^* k_g^2 k_g^{*^2} - \varepsilon_g^* k_{//,n}^2 K_n^2 - \varepsilon_1^* k_g^2 K_n^{*^2})]\end{aligned} \tag{46}$$

$$\begin{aligned}T_{4,n} =& \frac{p_{1,n}p_{2,n}^2}{p_{2,n}^2 - p_{1,n}^2}[-k_{//,n}^2(h_{1,n}^* + h_{2,n}^*)(k_g^{*^2}K_n^2 + k_g^2 K_n^{*^2}) \\ &+ 2j\varepsilon_0\omega k_{//,n}(-\varepsilon_g^* k_{//,n}^2 k_g^2 + \varepsilon_g^* k_g^{*^2}K_n^2 - \varepsilon_1^* K_n^2 K_n^{*^2}) \\ &- j\mu_0\omega k_{//,n}(k_g^{*^2}k_g^2 + K_n^2 K_n^{*^2}) \\ &\cdot(h_{1,n}h_{1,n}^* + h_{2,n}h_{2,n}^*) \\ &- k^2(\varepsilon_g^* k_g^2 k_g^{*^2} - \varepsilon_g^* k_{//,n}^2 K_n^2 - \varepsilon_1^* k_g^2 K_n^{*^2}) \\ &\cdot(h_{1,n} + h_{2,n})]\end{aligned} \tag{47}$$

$$
\begin{aligned}
T_{5,n} = &\frac{p_{1,n}p_{2,n}^2}{p_{2,n}^2 - p_{1,n}^2}[-k_{//,n}^2(h_{1,n}^* + h_{2,n}^*)(k_g^{*^2}K_n^2 + k_g^2K_n^{*^2}) \\
&+ 2j\varepsilon_0\omega k_{//,n}(-\varepsilon_g^* k_{//,n}^2 k_g^2 + \varepsilon_g^* k_g^{*^2} K_n^2 - \varepsilon_1^* K_n^2 K_n^{*^2}) \\
&- j\mu_0\omega k_{//,n}(k_g^{*^2} k_g^2 + K_n^2 K_n^{*^2}) \\
&\cdot (h_{1,n}h_{1,n}^* + h_{2,n}h_{2,n}^*) - k^2(\varepsilon_g^* k_g^2 k_g^{*^2} - \varepsilon_g^* k_{//,n}^2 K_n^2 \\
&- \varepsilon_1^* k_g^2 K_n^{*^2})(h_{1,n} + h_{2,n})] \qquad (48)
\end{aligned}
$$

$$
\begin{aligned}
T_{6,n} = &-\frac{p_{1,n}^2}{2}[-k_{//,n}^2 h_{2,n}^*(k_g^{*^2}K_n^2 + k_g^2K_n^{*^2}) \\
&+ j\varepsilon_0\omega k_{//,n}(-\varepsilon_g^* k_{//,n}^2 k_g^2 + \varepsilon_g^* k_g^{*^2} K_n^2 - \varepsilon_1^* K_n^2 K_n^{*^2}) \\
&- j\mu_0\omega k_{//,n} h_{1,n} h_{2,n}^*(k_g^{*^2} k_g^2 + K_n^2 K_n^{*^2}) \\
&- k^2 h_{1,n}(\varepsilon_g^* k_g^2 k_g^{*^2} - \varepsilon_g^* k_{//,n}^2 K_n^2 - \varepsilon_1^* k_g^2 K_n^{*^2})] \qquad (49)
\end{aligned}
$$

$$
\begin{aligned}
T_{7,n} = &-\frac{p_{2,n}^2}{2}[-k_{//,n}^2 h_{1,n}^*(k_g^{*^2}K_n^2 + k_g^2K_n^{*^2}) \\
&+ j\varepsilon_0\omega k_{//,n}(-\varepsilon_g^* k_{//,n}^2 k_g^2 + \varepsilon_g^* k_g^{*^2} K_n^2 - \varepsilon_1^* K_n^2 K_n^{*^2}) \\
&- j\mu_0\omega k_{//,n} h_{1,n}^* h_{2,n}(k_g^{*^2} k_g^2 + K_n^2 K_n^{*^2}) \\
&- k^2 h_{2,n}(\varepsilon_g^* k_g^2 k_g^{*^2} - \varepsilon_g^* k_{//,n}^2 K_n^2 - \varepsilon_1^* k_g^2 K_n^{*^2})]. \qquad (50)
\end{aligned}
$$

The sign of * denotes the conjugate complex number.

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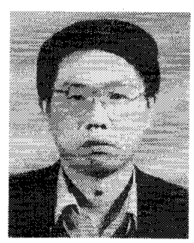

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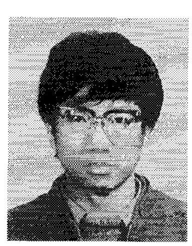

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