Dynamics of Uniform Vortex Patch With a Point Vortex

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***Abstract*—Vortex dynamics in two-dimensional inviscid, incompressible fluids are isomorphic to the dynamics of strongly magnetized guiding center plasmas restricted to $\mathbf{E}\times\mathbf{B}$ motion. In this paper, we exploit this analogy and study the dynamics of a uniform vortex patch with a point-like vortex using a particle-in-cell code. While the results show qualitative agreement with previous works in the linear regime when the normalized point vortex charge Γ is *small*, the dynamics shows many new features in the nonlinear regime: the point vortex "fangs" out the patch and moves toward the center, wrapping the patch around itself while setting up regions of zero vorticity as it moves.**

***Index Terms*—Nonneutral plasmas, PIC simulations, vortex dynamics.**

OF LATE, the realization that inviscid, incompressible, two-dimensional (2-D) Euler flows are isomorphic to 2-D magnetized guiding center plasmas has opened up many vistas in experimentally studying vortex dynamics [1]. Whereas conventional fluid dynamics is marred with experimental difficulties in realizing ideal 2-D, inviscid conditions with true free slip boundaries, such conditions are most naturally met in experiments of single species nonneutral plasmas (NNP) [1].

In the present work, we simulate the phenomenon of merger of two like-signed vorticity patches confined in a long, circular conducting cylinder using a 2-D, electrostatic particle-in-cell (PIC) code, XPDC2 [2]. In particular, we focus on a simple, yet interesting case of a central vortex patch of uniform vorticity and a point-like intense vortex both enclosed in a circular, grounded cylinder. We assume that the dynamics is restricted to the $r-\theta$ plane. The entire study is parametrized by Γ, the ratio of strengths (total charge) of point vortex to patch vortex. Moreover, densities of vortices are chosen so that certain shear conditions [3] are satisfied.

We begin by first setting Γ to be small and reproduce an already known work [3] qualitatively. Perturbed surface waves excited by the point vortex on the patch are of small amplitude. The central patch remains *nearly* circular throughout the dynamics. In Fig. 1, we present the results for two cases: 1) $\Gamma \simeq 6.42\times10^{-3} < \Gamma_c$, and 2) $\Gamma \simeq 1.605\times10^{2} > \Gamma_c$. Here, Γ_c is certain critical strength for the merger [3]. In Fig. 1(a), (b), we have shown the perturbed electrostatic energy $\delta W/W_0$ measured at $r=r_{\rm pa}$ as a function of normalized time for cases of oscillations and merger, respectively. In Fig. 1(c), the initial state of the system is shown, and in Fig. 1(d) the state for $\Gamma = 1.6\times10^{-2}$ during the merger is shown. These plots represent contours of *energy density* $qn(\mathbf{r},t)\varphi(\mathbf{r},t)/2$. (For presentation purposes, patch and point vortices are separately normalized to their initial peak energy density values.) The result is qualitatively in agreement with those obtained in [3]. We attribute the quantitative differences from previous work(s) to the presence of the conducting wall.

Next, we focus on a novel limit of large Γ ($\simeq$0.31), wherein, the interaction between point vortex and the patch vortex is strong throughout the evolution. Note that the physical picture discussed above and in [3] would be inapplicable here. As can be expected, the merger is rather rapid. Moreover, the point vortex "winds up" the patch and opens up the patch completely (see Fig. 2). For the figure shown (Fig. 2), we have used a 100×256 grid in the $r-\theta$ plane. About 22 000 simulation particles and 7000 simulation particles are used for the patch and *point* vortex, respectively. To begin with, the *point vortex* is of single grid size ($\Delta r \simeq 1\times10^{-3}$ m, $\Delta\theta \simeq 2.5\times10^{-3}$ rad) with a density of $n_{\rm pv} = 2\times10^{14}$ m^{-3}, located at $r \simeq 8\times10^{-2}$ m, and the patch vortex density $n_{\rm pa} = 2\times10^{12}$ m^{-3} with a radius of 6×10^{-2} m. For these parameters, $\Gamma \simeq 0.31$. Due to the size of the patch vortex and the bounding cylinder, the image vorticity plays a prime role throughout the dynamics. During evolution, due to the mutual velocity shear generated by the vortices, the point and the patch vortices alter their density distribution. In particular, the point vortex forms an inner "core" with an increased density and an outer diffused region which is partially smeared out. For example, for the figure shown (Fig. 2(f)), $n_{\rm pv}(\tau=3) \simeq 3.1\times10^{14}$ m^{-3} at the core while the average density of the patch vortex reduces.

Note also the region of zero vorticity in the immediate vicinity to the point vortex in Fig. 2(d)–(f). We speculate that eventually this irrotational region would be "expelled out" due to the usual diocotron or Kelvin-Helmholtz instability, in the process, creating trapped "vortex $\leftrightarrow$ antivortex" domains at later times before expulsion. Thus, this rather simple model, exhibits important features of the complex merger phenomena so ubiquitous in 2-D plasma and hydrodynamic turbulence.

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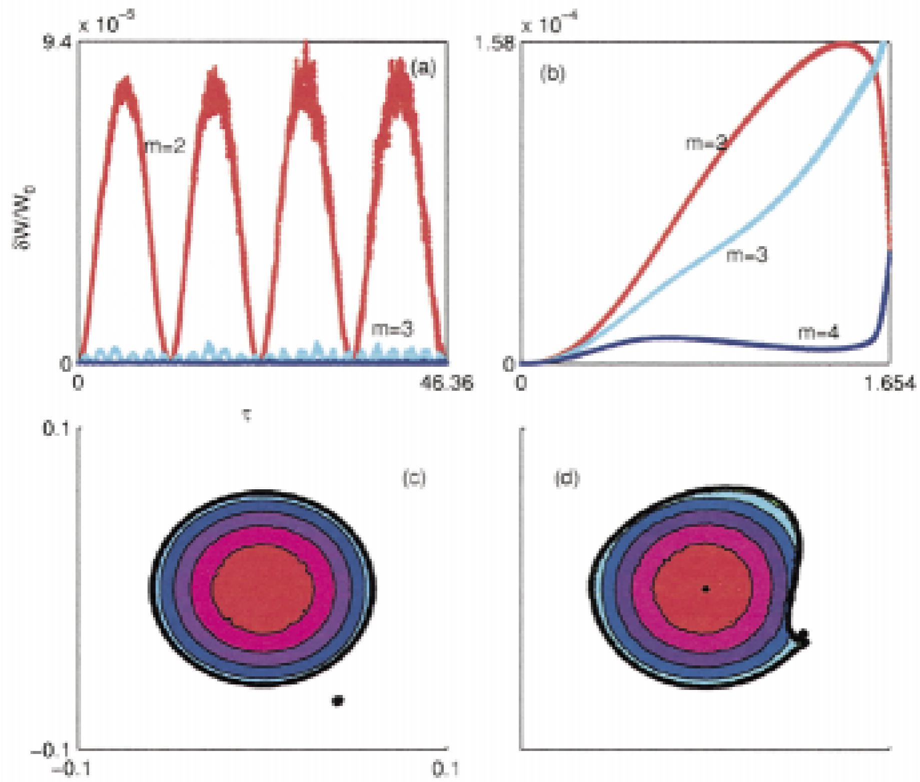

Fig. 1. (a) For $n_{\rm pv} = 4 \times 10^{12}$ m^{-3}($\Gamma = 6.42 \times 10^{-3}$), where $n_{\rm pv}$ is the point vortex density and $\tau = t/t_D$. (b) $n_{\rm pv} = 2 \times 10^{13}$ m^{-3} ($\Gamma = 1.605 \times 10^{-2}$). (c) Initial Configuration: normalized energy density contours in false color. In a scale of 0–1, "red" represents maximum value and "magenta" represents the minimum. (d) State during merger ($\tau = 1.654$). $B_o = 0.01$ T, $n_{\rm pa} = 2.0 \times 10^{12}$ m^{-3}, $R_w = 0.1$ m, $r_{\rm pa} = 6 \times 10^{-2}$ m, $r_{\rm pv} = 3 \times 10^{-3}$ m, $t_D = 3.477 \times 10^{-6}$ s, where t_D is the diocotron time or patch turn-over time and $W_0 = 9.07 \times 10^{-8}$ J is the total electrostatic energy of the unperturbed system.

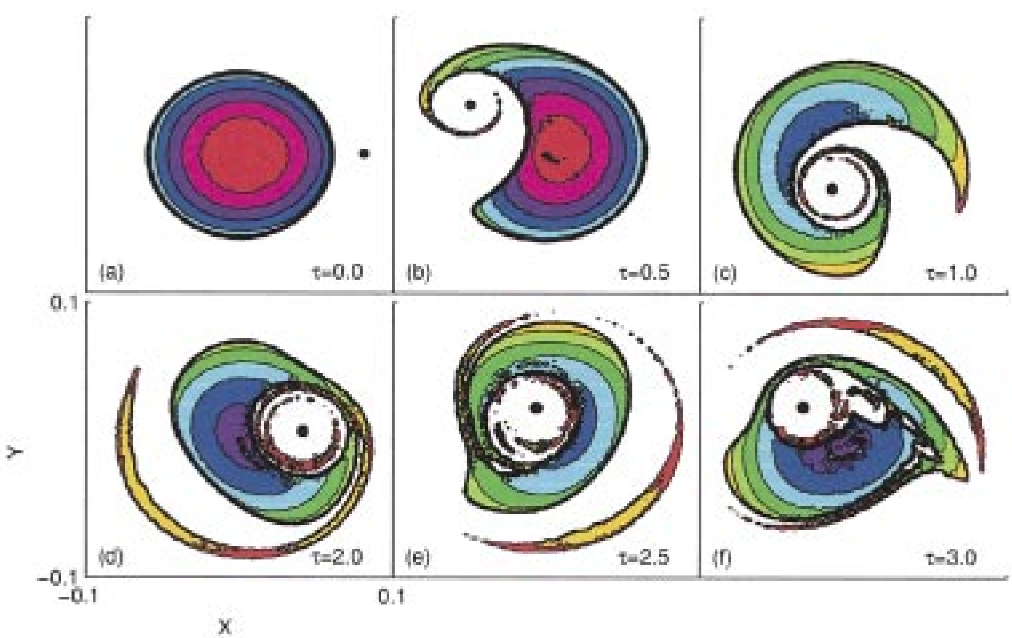

Fig. 2. Normalized energy density contours in false color: (a) initial condition at $\tau = t/t_D = 0$. For clarity, the conducting wall $r = 0.1$ m is not shown. (b) $\tau = 0.5$, (c) $\tau = 1$, (d) $\tau = 2$, (e) $\tau = 2.5$, (f) $\tau = 3$. All parameters are the same as Fig. 1 except $n_{\rm pv} = 2.0 \times 10^{14}$ m^{-3} so that $\Gamma(\simeq 0.31)$ is in the nonlinear regime.