Chap 1-2

1-2) Phenomenons not adequately described by circuit theory →

1) Microwave Oven working phenomenon.

2) Mobile phones

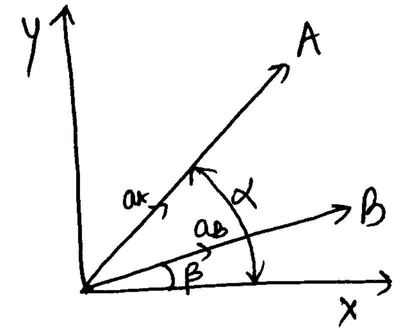
1-5) Fundamental quantities of EM Model →

Name	Units
E : Electric field Intensity	V/m
D : Electric flux Density	C/M^2
B : Magnetic flux Density	T
H : Magnetic field Intensity	A/m

1-7) Source quantities for electromagnetic model →

- Charge or distribution of charges form the source quantity.
- Charge generated current for the source for magnetic model.

2-9)



a_A can be resolved into $x + y$ components as —

$a_A = \cos\alpha\, a_x + \sin\alpha\, a_y$

$a_B = \cos\beta\, a_x + \sin\beta\, a_y$

$\theta_{AB} = \alpha - \beta$

$$a_A \cdot a_B = \frac{|\bar{A}| \cdot |\bar{B}|}{|\bar{A}|\,|\bar{B}|} \cos\theta_{AB} = (\cos\alpha\, a_x + \sin\alpha\, a_y) \cdot (\cos\beta\, a_x + \sin\beta\, a_y)$$

$$\Rightarrow \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

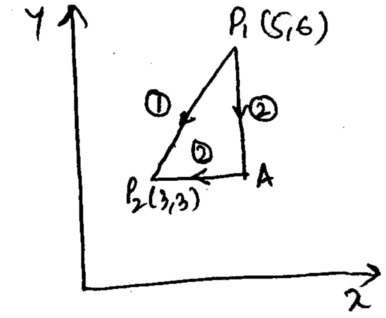
III^ly

$$a_A \times a_B = \frac{|\bar{A}|\,|\bar{B}|}{|\bar{A}|\,|\bar{B}|} \sin\theta_{AB}(-a_z) = (\cos\alpha\, a_x + \sin\alpha\, a_y) \times (\cos\beta\, a_x + \sin\beta\, a_y)$$

$$\sin(\alpha-\beta)\cancel{(-a_z)} = (\sin\alpha\cos\beta - \cos\alpha\sin\beta)\cancel{(-a_z)}$$

$$\therefore \sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

2-20)



Given $F = a_x xy + a_y(3x - y^2)$

a) along direct path P_1P_2.

equation of line passing through P_1P_2 is —

$$y-6 = \left(\frac{6-3}{5-3}\right)(x-5)$$

$$2(y-6) = 3(x-5)$$

$$y = \frac{3}{2}(x-1) \qquad x = \frac{2}{3}y + 1$$

a) $$\therefore \int F.dl = \int_5^3 x \cdot \frac{3}{2}(x-1)\,dx + \int_6^3 (2y+3-y^2)\,dy$$

$$= \frac{3}{2}\left\{ \frac{1}{3}x^3\Big|_5^3 - \frac{x^2}{2}\Big|_5^3 \right\} + \left\{ 2\frac{y^2}{2}\Big|_6^3 + 3y\Big|_6^3 - \frac{y^3}{3}\Big|_6^3 \right\}$$

$$= \frac{3}{2}\{-32.67 + 8\} + \{-27 - 9 + 63\}$$

$$= \{-37\} \qquad + \{27\}$$

$$\boxed{\int F.dl = -10}$$

b) i) P_1 to A ie $(5,6)$ to $(5,3)$

$$\int F.dl_1 = \int_6^3 (3x - y^2)\,dy\Big|_{x=5} = 15y\Big|_6^3 - \frac{y^3}{3}\Big|_6^3 = 15(-3) - \frac{1}{3}(27-216)$$

$$\int F.dl_1 = 18$$

(ii) A to P_2 ie $(5,3)$ to $(3,3)$

$$\int F.dl_2 = \int_5^3 xy\,dx\Big|_{y=3} = 3\frac{x^2}{2}\Big|_5^3 = \frac{3}{2}(9-25) = -24$$

$$\int F.dl = \int F.dl_1 + \int F.dl_2 = 18 - 24 = \underline{\underline{-6}}$$

2-24) Evaluate $\oint_S (a_R 3 \sin\theta) \cdot ds$ for $R=5$ at origin $(0,0,0)$

ds for R const. $= R^2 \sin\theta \, d\theta \, d\phi \, a_R$

$$\oint_S (a_R \, 3\sin\theta) \, ds = \oint_S 3R^2 \sin^2\theta \, d\theta \, d\phi \Big|_{R=5}$$

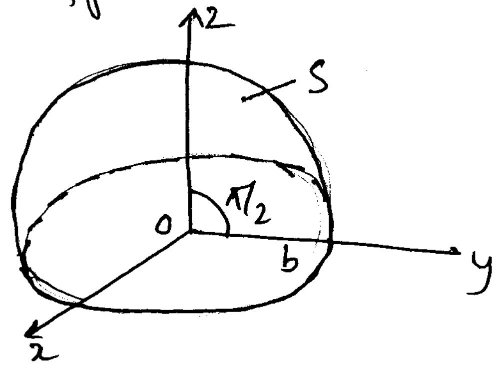
$$= 75 \int_0^{\pi} \sin^2\theta \, d\theta \int_0^{2\pi} d\phi$$

$$= 150\pi \int_0^{\pi} \left(\frac{1-\cos 2\theta}{2} \right) d\theta$$

$$= 150\pi \left[\pi/2 - \frac{\sin 2\theta}{4} \Big|_0^{\pi} \right]$$

$$= \underline{\underline{75\pi^2}}$$

2-36) $A = a_\phi (\sin\phi/2)$, verify stokes theorem over hemispherical surface.



Stokes theorem states — $\int_S (\nabla \times A) \cdot ds = \oint_C A \cdot dl$.

LHS $\longrightarrow \int_S (\nabla \times A) \, ds$

$$\nabla \times A = \frac{1}{R^2 \sin\theta} \begin{vmatrix} a_R & a_\theta R & a_\phi R \sin\theta \\ \partial/\partial R & \partial/\partial\theta & \partial/\partial\phi \\ A_R & RA_\theta & R\sin\theta A_\phi \end{vmatrix}$$

$$= \frac{1}{R^2 \sin\theta} \begin{vmatrix} a_R & R a_\theta & R\sin\theta \, a_\phi \\ \partial/\partial R & \partial/\partial\theta & \partial/\partial\phi \\ 0 & 0 & R\sin\theta \sin\phi/2 \end{vmatrix}$$

$$= \frac{1}{R^2 \sin\theta} \left[(R\cos\theta \sin\phi/2) a_R - (\sin\theta \sin\phi/2) a_\theta \right]$$

$$\nabla \times A = \frac{1}{R} \cot\theta \sin\phi/2 \, a_R - \frac{1}{R^2} \sin\phi/2 \, a_\theta .$$

For the given region, $dS = R^2 \sin\theta \, d\theta \, d\phi \, a_R$

$$\therefore \int_S (\nabla \times A) dS = \int_0^{\pi}\int_0^{2\pi} \frac{1}{R} \frac{\cos\theta}{\cancel{\sin\theta}} \sin\phi/2 \cdot R^2 \cancel{\sin\theta} \, d\theta \, d\phi \Big|_{R=b}$$

$$= b \int_0^{\pi/2} \cos\theta \, d\theta \int_0^{2\pi} \sin\phi/2 \, d\phi$$

$$\int_S (\nabla \times A) \cdot dS = 4b$$

RHS $\longrightarrow \oint_c A \cdot dl$

$$= \oint_c (a_\phi \sin\phi/2) \cdot (a_R \, dR + a_\theta R \, d\theta + a_\phi R\sin\theta \, d\phi)$$

$$= \oint_c R\sin\theta \sin\phi/2 \, d\phi \Big|_{\substack{R=b \\ \theta=\pi/2}}$$

$$= b \sin\pi/2 \int_0^{2\pi} \sin\phi/2 \, d\phi$$

$$\oint_c A \cdot dl = 4b$$

LHS = RHS

$\therefore$ Stokes theorem is verified for the given vector function A.